Assertion and Reason Questions on Exploring Algebraic Identities Chapter 4 for Class 9

Directions: In each question below, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:
(a) Both A and R are true, and R is the correct explanation of A.
(b) Both A and R are true, but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q.1
Assertion (A): The expression $(x + y)^2 = x^2 + 2xy + y^2$ is an identity.
Reason (R): An algebraic identity is an equation that holds true for all values of the variables appearing in it.
Answer: (a) Both A and R are true; R is the correct explanation of A.
Explanation: The definition given in R is correct. And $(x + y)^2 = x^2 + 2xy + y^2$ satisfies this completely. R is the very reason A is true, so (a).
Q.2
Assertion (A): $(5x + 2y)^2 = 25x^2 + 20xy + 4y^2$
Reason (R): $(a + b)^2 = a^2 + 2ab + b^2$, where a and b can represent any algebraic terms.
Answer: (a) Both A and R are true; R is the correct explanation of A.
Explanation: Substituting $a = 5x$ and $b = 2y$ into the identity in R: $(5x)^2 + 2(5x)(2y) + (2y)^2 = 25x^2 + 20xy + 4y^2$. This is A. R is the identity that makes A possible.
Q.3
Assertion (A): 43^2 can be calculated as $(40 + 3)^2 = 1600 + 240 + 9 = 1849$.
Reason (R): $(a - b)^2 = a^2 - 2ab + b^2$ is the identity used to find 43^2.
Answer: (c) A is true; R is false.
Explanation: A is correct. $43^2 = (40 + 3)^2$ is expanded using $(a + b)^2 = a^2 + 2ab + b^2$, giving 1849. However, R gives the wrong identity. Since the identity named in R is incorrect for this case, R is false. Option (c).
Q.4
Assertion (A): $x^2 + 4x + 4 = (x + 2)^2$

Reason (R): The identity $a^2 + 2ab + b^2 = (a + b)^2$ can be used in reverse to factorise algebraic expressions.
Answer: (a) Both A and R are true; R is the correct explanation of A.
Explanation: Recognising that $x^2 = (x)^2$, $4 = (2)^2$, and $4x = 2(x)(2)$ matches the pattern $a^2 + 2ab + b^2$ with $a = x$ and $b = 2$. R directly states that the identity can be used in reverse for factorisation which is what was done to get $(x + 2)^2$. Option (a).
Q.5
Assertion (A): $a^2 – b^2 = (a + b)(a – b)$
Reason (R): $(a + b)^2 – (a – b)^2 = 2ab$.
Answer: (c).A is true but R is false.
Explanation: A is an algebraic identity. $(a + b)^2 – (a – b)^2 = (a^2 + 2ab + b^2) – (a^2 – 2ab + b^2) = 4ab$. So R is false. That makes this option (c).
Q.6
Assertion (A): $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
Reason (R): This identity is derived by treating $(b + c)$ as a single term d, and applying $(a + d)^2 = a^2 + 2ad + d^2$, then expanding $d^2 = (b + c)^2$.
Answer: (a) Both A and R are true; R is the correct explanation of A.
Explanation: A is an identity. The method described in R, substituting $d = b + c$, expanding $(a + d)^2$ and then expanding $d^2 = (b + c)^2 = b^2 + 2bc + c^2$ is the derivation. R correctly and completely explains the origin of A. Option (a).
Q.7
Assertion (A): $50p^2 + 60pq + 18q^2 = 2(5p + 3q)^2$
Reason (R): Before applying an identity, it is sometimes necessary to first take out a common numerical factor from all terms of the expression.
Answer: (a) Both A and R are true; R is the correct explanation of A.
Explanation: Taking 2 as a common factor: $50p^2 + 60pq + 18q^2 = 2(25p^2 + 30pq + 9q^2)$. Then $25p^2 = (5p)^2$, $9q^2 = (3q)^2$, and $30pq = 2(5p)(3q)$, so $(5p + 3q)^2$. Hence the full expression $= 2(5p + 3q)^2$. A is correct. R describes this strategy of removing the common factor first, making R the explanation of A. Option (a).
Q.8
Assertion (A): $x^2 + 11x + 30 = (x + 5)(x + 6)$
Reason (R): To factorise $x^2 + bx + c$ into $(x + p)(x + q)$, we need $p + q = b$ and $p \times q = c$.
Answer: (a) Both A and R are true; R is the correct explanation of A.

Explanation: For $x^2 + 11x + 30$, we need $p + q = 11$ and $pq = 30$. Trying pairs of factors of 30: 5 and 6 give $5 + 6 = 11$ and $5 \times 6 = 30$. So the factors are $(x + 5)(x + 6)$, confirming A. R is the method that makes this work. Option (a).

Q.9

Assertion (A): $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Reason (R): $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ is obtained simply by replacing b with $-b$ in the expansion of $(a + b)^3$.

Answer: (b) Both A and R are true, but R does not explain A.

Explanation: A is the correct expansion of $(a + b)^3$. R is also a true and valid result. However, R does not explain why A is true. Option (b).

Q.10

Assertion (A): $x^2 - 5x + 6 = (x - 2)(x - 3)$

Reason (R): When the coefficient of x is negative in $x^2 + bx + c$, the values of p and q in $(x + p)(x + q)$ must both be negative.

Answer: (a) Both A and R are true; R is the correct explanation of A.

Explanation: For A: we need $p + q = -5$ and $pq = 6$.Taking $p = -2$ and $q = -3$: $(-2) + (-3) = -5$ and $(-2)(-3) = 6$. So $A = (x - 2)(x - 3)$ is correct. R correctly explains A. Option (a).

Q.11

Assertion (A): $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Reason (R): $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$, and the factorisation of $x^3 - y^3$ is obtained by replacing y with $-y$ in this identity.

Answer: (c) A is true; R is false.

Explanation: A is the correct factorisation. R, however, is incorrect. Replacing y with $-y$ in $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ gives $x^3 + (-y)^3 = x^3 - y^3 = (x - y)(x^2 + xy + y^2)$.The reasoning in R is flawed. R is false. Option (c).

Q.12

Assertion (A): If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$.

Reason (R): The identity $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$ holds for all real values of x, y, and z.

Answer: (a) Both A and R are true; R is the correct explanation of A.

Explanation: R is an identity. When we substitute $x + y + z = 0$ into the right-hand side of R, we get $0 \times (x^2 + y^2 + z^2 - xy - yz - zx) = 0$. So the left-hand side becomes $x^3 + y^3 + z^3 - 3xyz = 0$, which means $x^3 + y^3 + z^3 = 3xyz$. This is exactly what A states. R correctly explains A. Option (a).

Q.13

Assertion (A): The expression $p^3 + 6p^2q + 12pq^2 + 8q^3$ represents the volume of a cube whose side is $(p + 2q)$ units.

Reason (R): The volume of a cube of side a is a^3, so to find the side from a given volume expression, one must express the volume in the form $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.

Answer: (a) Both A and R are true; R is the correct explanation of A.

Explanation: Comparing $p^3 + 6p^2q + 12pq^2 + 8q^3$ with $a^3 + 3a^2b + 3ab^2 + b^3$: we see $a = p$ and $b = 2q$, since $3a^2b = 3p^2(2q) = 6p^2q$, $3ab^2 = 3p(4q^2) = 12pq^2$, and $b^3 = 8q^3$. So the expression $= (p + 2q)^3$ confirms A. R correctly describes the process of reverse-matching using the cube identity. Option (a).

Q.14

Assertion (A): $(x^2 – 7x + 12) ÷ (5x^2 + 5x – 100)$ simplifies to $(x – 3) ÷ [5(x + 5)]$, provided the denominator is not zero.

Reason (R): The common factor $(x – 4)$ can be cancelled from the numerator and denominator only when it is confirmed to be non-zero, which follows from the condition $5x^2 + 5x – 100 \neq 0$.

Answer: (a) Both A and R are true; R is the correct explanation of A.

Explanation: Factorising: numerator $(x−3)(x−4)$, denominator $5(x−4)(x+5)$. Cancel $(x−4)$ since denominator $\neq 0$ ensures it isn't zero. The result is $(x – 3) / [5(x + 5)]$. A is correct, and R gives the justification for the cancellation step. Option (a).

Q.15

Assertion (A): For any three consecutive integers, if we add the smallest and largest squares and then subtract twice the middle square, the result is always 2.

Reason (R): If the three consecutive integers are $(n – 1)$, n, and $(n + 1)$, then $(n – 1)^2 + (n + 1)^2 – 2n^2 = 2$, which follows from expanding using the identities $(a – b)^2$ and $(a + b)^2$.

Answer: (a) Both A and R are true; R is the correct explanation of A.

Explanation: Expanding R step by step: $(n – 1)^2 = n^2 – 2n + 1$ and $(n + 1)^2 = n^2 + 2n + 1$. Their sum $= 2n^2 + 2$. Subtracting $2n^2$ gives exactly 2. So R correctly explains why A is always true, making option (a) correct.