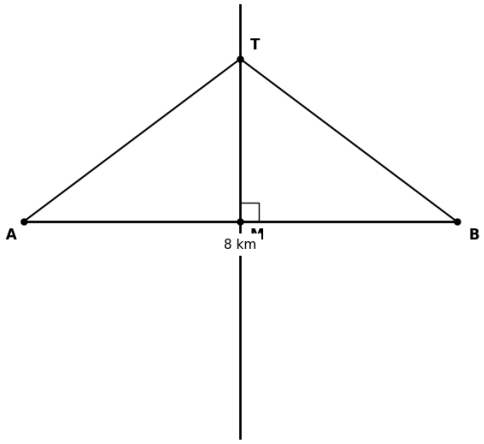
Case Study Chapter 5: I'm Up and Down and Round and Round for Class 9

Case Study 1: The Telecom Tower

Read the passage and answer the questions below.

Two villages, A and B, are 8 km apart. A telecom company wants to set up a single mobile tower that is equally distant from both villages, so that signal strength is balanced. The engineers know the tower's location must lie somewhere along a particular line related to A and B.



Q1. On which line must every possible location for the tower lie?
(a) The line segment AB itself
(b) The perpendicular bisector of AB
(c) A line parallel to AB
(d) Any random line
Q2. If the tower is placed exactly at the midpoint of AB, what is the smallest possible ‘equal distance’ (radius) it could have to both villages?
Q3. If the engineers move the tower's location further away from the midpoint, but still along the perpendicular bisector, what happens to this ‘equal distance’ value?
Q4. Could this equal distance value ever be less than 4 km?
Q5. Suppose the tower is eventually placed at a point where this equal distance works out to exactly 5 km. How far is the tower from the midpoint M of AB, measured along the perpendicular bisector?

Solutions:

Q1. (b)

The set of all points equidistant from two fixed points A and B is exactly the perpendicular bisector of segment AB

Q2. The smallest circle passing through both A and B has AB itself as its diameter, so the radius is half of AB: 8 km ÷ 2 = 4 km. No smaller circle can touch both villages.

Q3. It increases.

As the centre moves further from the midpoint along the perpendicular bisector, its distance to both A and B grows, so the radius of the circle through A and B keeps increasing.

Q4. No, 4 km is the minimum possible value. 4 km, achieved only at the midpoint of AB, is the absolute minimum. Every other point on the perpendicular bisector gives a distance strictly greater than 4 km.

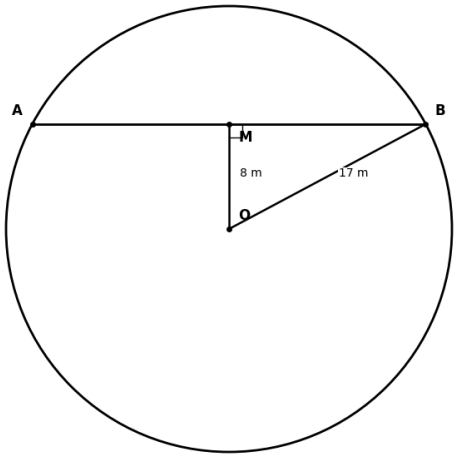
Q5. 3 km

The tower, the midpoint M, and village A form a right triangle, with the tower-to-A distance (5 km) as the hypotenuse and half of AB (4 km) as one leg. By the Baudhāyana–Pythagoras theorem: $5^2 = 4^2 + (\text{distance})^2$, so $(\text{distance})^2 = 25 - 16 = 9$, giving 3 km.

Case Study 2: The Circular Park Path

Read the passage and answer the questions below.

A circular park has a fountain at its exact centre, O, and a radius of 17 m. A straight paved path cuts across the park as a chord, and the perpendicular distance from the fountain to this path is measured as 8 m.



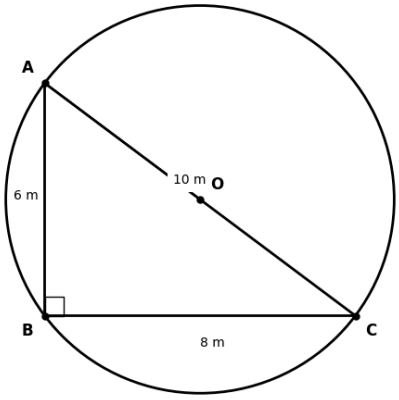
Q1. What is the length of the paved path?
Q2. A second path is laid only 6 m from the fountain. Compared to the first path (30 m), will this second path be longer or shorter?
Q3. What is the maximum possible length any straight path through this park could have?
Q4. Which mathematical result did you use to calculate the path length in Q1?
Q5. Suppose a third path is laid at a perpendicular distance of exactly 17 m from the fountain. What does this path look like?

Solutions:
Q1. Using $2\sqrt{(r^2 - d^2)}$: half the path's length is $\sqrt{(17^2 - 8^2)} = \sqrt{(289 - 64)} = \sqrt{225} =$ 15 m. Doubling this gives the full chord length, 30 m.
Q2. The closer a chord is to the centre, the longer it is. Since 6 m is less than 8 m, this second path is closer to the fountain and must therefore be longer than 30 m.
Q3. The longest possible chord in any circle is its diameter, equal to twice the radius. Here, 2×17 m = 34 m, achieved only by a path passing exactly through the fountain.
Q4. You first use the theorem that the perpendicular from the centre bisects the chord, turning the problem into a right triangle, and then apply the Baudhāyana - Pythagoras theorem to that right triangle.
Q5. When the perpendicular distance equals the radius itself, the "chord" has shrunk down to a single point on the boundary, there's no path left to pave, since the chord length $2\sqrt{(17^2 - 17^2)}$ works out to zero.

Case Study 3: Three Tubewells in a Field

Read the passage and answer the questions below.

A farmer has three tubewells at points A, B and C in his field, and the three points are not in a straight line. He wants to build a single circular water tank whose boundary touches all three tubewells, so that one tank can supply all of them equally. The distances he measures are AB = 6 m, BC = 8 m, and AC = 10 m, with the right angle of the field falling exactly at B.



Q1. Why can the farmer always be sure that a circle exists which passes through all three tubewells, A, B and C?

Q2. How should the farmer locate the exact centre of this circular tank?

(a) At the midpoint of AB only

(b) Anywhere inside the triangle

(c) At the point where the perpendicular bisectors of any two sides meet

(d) At the longest side's midpoint

Q3. Since $AB^2 + BC^2 = 36 + 64 = 100 = AC^2$, the triangle is right-angled at B. Where will the circumcentre (and therefore the tank) be located?

Q4. The radius of the circular tank will be _____ m.

Q5. True or False

Statement: If the farmer later discovers that A, B and C are actually all in a straight line, he can still build a single circular tank touching all three tubewells.

Solutions:

Q1. Because A, B and C are not collinear. The theorem that three non-collinear points always lie on exactly one circle guarantees a unique circumcircle through all three. If the points were in a straight line, no such circle could exist.

Q2. (c)

The centre of the circle through A, B and C. The circumcentre always lies at the point where the perpendicular bisectors of the triangle's sides intersect, since this is the only point equidistant from all three vertices.

Q3. Midpoint of AC

For any right-angled triangle, the circumcentre always lands exactly at the midpoint of the hypotenuse.

Q4. Because the circumcentre sits at the midpoint of the hypotenuse, the circumradius is simply half of AC, which is half of 10 m, giving 5 m.

Q5. False

A circle can never pass through three collinear points, because the perpendicular bisectors of the segments joining them would be parallel and never meet, so there's no possible centre.