Case Study Chapter 6: Measuring Space: Perimeter and Area for Class 9

Case Study 1: The School Garden

The Class 9 students of Sunrise Public School are helping redesign the school garden. The garden has two sections. Section A is a triangular patch with sides 9 m, 40 m, and 41 m. Section B is a circular flower bed of radius 14 m, out of which a sector with a central angle of 90° is to be planted with roses. The rest of the circle will have grass. The school plans to put a decorative border along the entire boundary of the rose sector (including the two radii and the arc).

Q.1 What is the semi-perimeter of the triangular Section A?
(a) 30 m (b) 40 m (c) 45 m (d) 90 m
Q.2 Check whether Section A is a right-angled triangle.
Q.3 Find the area of Section A.
Q.4 Find the area of the rose sector (Section B) and the length of the decorative border around it. (Use $\pi = 22/7$)
Q.5 If the decorative border costs ₹120 per metre and the grass area (the remaining three-quarters of the circle) needs to be seeded at ₹15 per m^2, find the total cost for both. (Use $\pi = 22/7$)

Solutions:

Q.1: (c) 45 m
$s = (9 + 40 + 41)/2 = 90/2 = 45$ m

Q.2: $9^2 + 40^2 = 81 + 1600 = 1681 = 41^2$
Yes, Section A is a right-angled triangle (right angle between the 9 m and 40 m sides).

Q.3: Since it is a right-angled triangle:
Area = $\frac{1}{2} \times 9 \times 40 = 180$ m^2

Q.4: Radius = 14 m, $\theta = 90°$
Area of rose sector:
$= \pi r^2 \times \theta/360$

$= (22/7) \times 196 \times 90/360$
$= (22/7) \times 196 \times 1/4$
$= 22 \times 7 = 154\ m^2$

Perimeter of the rose sector (border):
Arc length $= 2\pi r \times \theta/360 = 2 \times (22/7) \times 14 \times 1/4 = 22$ m
Perimeter = arc + 2 radii = 22 + 14 + 14 = 50 m

Q.5: Cost of decorative border:
Perimeter of rose sector = 50 m
Cost = 50 × ₹120 = ₹6,000

Area of grass (three-quarter circle):
Full circle area $= \pi r^2 = (22/7) \times 196 = 616\ m^2$
Grass area $= 616 - 154 = 462\ m^2$

Cost of seeding grass:
= 462 × ₹15 = ₹6,930

Total cost = ₹6,000 + ₹6,930 = ₹12,930

Case Study 2: The 400 m Athletics Track

The school's annual sports day is around the corner, and the sports committee is designing a 400 m athletics track. The track has two straight sections of length 84.39 m each and two curved portions that are semicircles with a common centre. The innermost semicircle on each side has a radius of 36.5 m. Each lane is 1.22 m wide. An athlete in lane 1 runs at a distance of 0.3 m from the inner border. The two semicircles together make up one complete circle.

Q.1 What is the total length of the two straight sections of the track?
(a) 84.39 m (b) 96.78 m (c) 168.78 m (d) 184.39 m
Q.2 The athlete in lane 1 runs 0.3 m from the inner border. What is the effective radius of the circular path she traces on the curved portions?

Q.3 Using $\pi \approx 3.1416$, find the total length of the curved portions (the two semicircles combined) that the lane 1 athlete runs.
Q.4 Verify that the total distance run by the lane 1 athlete in one complete circuit is 400 m.
Q.5 The lane 2 runner's semicircles have a radius of $36.5 + 1.22 + 0.3 = 38.02$ m. Without the stagger, how much extra distance would she run compared to the lane 1 athlete? (Use $\pi \approx 3.1416$)
Solutions:
Q.1: (c) 168.78 m
Two straight sections: $84.39 \times 2 = 168.78$ m.

Q.2: $36.5 + 0.3 = 36.8$ m

Q.3: The two semicircles together form one complete circle of radius 36.8 m.
Circumference = $2 \times 3.1416 \times 36.8$
$= 2 \times 3.1416 \times 36.8$
$= 231.22$ m

Q.4: Straight sections = 168.78 m
Curved sections = 231.22 m
Total = $168.78 + 231.22 = 400$ m

Q.5: Circumference for lane 2 curves = $2 \times 3.1416 \times 38.02 = 238.89$ m (approx.)
Circumference for lane 1 curves = 231.22 m
Extra distance = $238.89 - 231.22 = \approx 7.67$ m
This is the stagger that must be given to the lane 2 runner at the start.

Case Study 3: The Semicircle Paths Paradox
Two friends, Aryan and Maya, are looking at a geometry diagram. There are two paths connecting points P and Q. Path 1 is a single large semicircle with PQ as its diameter. Path 2 is made up of three smaller semicircles arranged along PQ, alternating above and below the straight line PQ, collectively spanning the same distance PQ. Aryan is convinced Path 1 is longer. Maya says Path 2 must be longer since it ‘wiggles’. Their maths teacher says they are both wrong.

Q.1 If PQ = 2a units, what is the radius of the large semicircle in Path 1?
Q.2 Write the expression for the length of Path 1.
Q.3 The three semicircles in Path 2 have radii b', c', and d' respectively, and they span the full length PQ. Express the relationship between a, b', c', and d'.
Q.4 Find the total length of Path 2 and compare it with Path 1.
Q.5 Explain in your own words why this result is surprising, and what mathematical principle makes it work.

Solutions:

Q.1: PQ is the diameter, so radius = PQ/2 = a units.

Q.2:
Length = $\frac{1}{2} \times$ circumference = $\frac{1}{2} \times 2\pi a = \pi a$ units

Q.3: The diameters of the three semicircles together equal PQ:
$2b' + 2c' + 2d' = 2a$
Therefore: $a = b' + c' + d'$

Q.4: Length of Path 2 = $\pi b' + \pi c' + \pi d' = \pi(b' + c' + d')$
Since $a = b' + c' + d'$:
Length of Path 2 = πa

Path 1 length = πa = Path 2 length.
Both paths are equal in length.

Q.5 It is surprising because Path 2 'wiggles' above and below the line, which intuitively feels like it should cover more distance. But the key is that each semicircle's length depends only on its diameter, and the sum of the diameters of the three smaller semicircles always equals the diameter of the large semicircle (since they together span PQ). Since arc length is proportional to diameter for semicircles, the total lengths are always equal. The result holds even if you use 4, 5, or any number of smaller semicircles, as long as their diameters sum to PQ.