# Case Study Class 10 Chapter 1: Real Numbers

## Case Study 1: The Prime Relay

**A number passed hand to hand**

A teacher starts a class game by announcing the number 2. The first student multiplies it by any prime number and passes the result on; the next student does the same with whatever number they receive, and so on down the row. After several hands, the last student announces the number 173250.

**Q1: The full prime factorisation of 173250 is 2 × ?**

Solution: Dividing repeatedly by small primes: $173250 = 2 \times 86625 = 2 \times 3^2 \times 9625 = 2 \times 3^2 \times 5^3 \times 77 = 2 \times 3^2 \times 5^3 \times 7 \times 11$. So the blank is $3^2 \times 5^3 \times 7 \times 11$.

**Q2: How many students (after the teacher) took part in the relay?**

(a) 5 (b) 6

(c) 7 (d) 8

Solution: Counting every prime factor with repetition: one 2, two 3s, three 5s, one 7, one 11, that's 8 prime factors in total. The teacher's starting number already accounts for the first '2,' so the remaining 7 factors were each contributed by one student.

**Q3: Which prime did the most students use, and how many times?**

Solution: 5 appears with power 3 in the factorisation, more than any other prime, so 5 was used three times, by three different students.

**Q4: Excluding the teacher's starting 2, the smallest and largest primes used by students were:**

(a) 2 and 11

(b) 3 and 11

(c) 3 and 7

(d) 2 and 7

Solution: The primes the students multiplied in were 3, 3, 5, 5, 5, 7 and 11. The smallest is 3 and the largest is 11.

## Case Study 2: The Morning Lap

**Two runners, one circular track**

Sonia and Ravi jog every morning around the same circular track. Sonia takes 18 minutes to complete one full lap; Ravi, a touch faster on the bends, takes 12 minutes. They start together at the gate and run in the same direction.

**Q1: Expressing both lap times as products of primes: 18 = ? and 12 = ?**

Solution: $18 = 2 \times 3^2$ and $12 = 2^2 \times 3$.

**Q2: True or False: They will both be back at the gate together after the HCF of 18 and 12, i.e. 6 minutes.**

(a) True

(b) False

Solution: False. HCF only tells you the largest time interval that divides both lap durations evenly, it has nothing to do with when the two runners actually line up again at the gate. LCM can be usd to find that.

**Q3: After how many minutes will Sonia and Ravi next be together at the gate?**

(a) 6  (b) 30

(c) 36  (d) 72

Solution: LCM(18, 12) = $2^2 \times 3^2 = 4 \times 9 = 36$ minutes.

**Q4: In one line, why does LCM and not HCF decide when two repeating events coincide again?**

Solution: Because the moment they meet again has to be a common multiple of both lap times, a time both cycles land on exactly, and the soonest such moment is, by definition, the least common multiple.