Case Study Class 10 Chapter 2: Polynomials

Case Study 1: The Suspension Bridge

A civil engineer is designing a suspension bridge. The main cable hangs in the shape of a parabola between two towers. The engineer models the cable using the quadratic polynomial $p(x) = x^2 - 6x + 8$, where x is the horizontal distance (in metres) from the left tower and p(x) is the height of the cable above the road. The road is represented by the x-axis.

Questions and Answers:

Q1. The zeros of $p(x) = x^2 - 6x + 8$ represent:

a) Heights of the towers

b) Points where the cable touches the road

c) The width of the river

d) Midpoint of the bridge

Solution: b) Points where the cable touches the road

The zeros of p(x) are the values of x where $p(x) = 0$, i.e. where the cable height above the road is zero: the points where the cable meets the road level.

Q2. What are the zeros of $p(x) = x^2 - 6x + 8$?

a) $x = 2$ and $x = 4$

b) $x = -2$ and $x = -4$

c) $x = 1$ and $x = 8$

d) $x = 3$ and $x = 3$

Solution: Factorise: $x^2 - 6x + 8 = (x - 2)(x - 4)$. Setting each factor to zero gives $x = 2$ and $x = 4$.

Q3. The sum of the zeros of $p(x) = x^2 - 6x + 8$ using the coefficient formula is:

a) −8 b) 8

c) 6   d) −6

Solution: Sum of zeros = −(coefficient of x) / (coefficient of x^2) = $-(-6)/1 = 6$.

Verify: $2 + 4 = 6$

Q4. The graph of $p(x) = x^2 - 6x + 8$ opens:

a) Downward, since $a < 0$

b) Upward, since $a > 0$

c) Sideways

d) Cannot be determined

Solution: The leading coefficient $a = 1 > 0$, so the parabola opens upward, like a U-shape.

Q5. The product of the zeros of $p(x) = x^2 - 6x + 8$ is:

a) −6 b) 6

c) 8 d) −8

Solution: Product of zeros = $c/a = 8/1 = 8$. Verify: $2 \times 4 = 8$

Case Study 2: The Satellite Dish

A satellite dish manufacturer designs dishes whose cross-sectional shape follows a quadratic polynomial. The cross-section of one dish is given by $p(x) = 2x^2 - 8$, where x is measured in metres. A quality inspector needs to verify the properties of this polynomial before the dish goes into production. The dish is symmetric about the vertical axis ($x = 0$).

Questions and Answers:

Q1. The zeros of $p(x) = 2x^2 - 8$ are:

a) $x = 2$ only

b) $x = -2$ and $x = 2$

c) $x = 4$ and $x = -4$

d) $x = 0$ and $x = 4$

Solution: Set $2x^2 - 8 = 0 \rightarrow 2x^2 = 8 \rightarrow x^2 = 4 \rightarrow x = \pm 2$. So zeros are $x = -2$ and $x = 2$, confirming the dish's symmetric design.

Q2. The sum of the zeros of $p(x) = 2x^2 - 8$ using the coefficient formula is:

a) −4 b) 4

c) 0 d) 8

Solution: $p(x) = 2x^2 + 0 \cdot x - 8$, so $b = 0$. Sum $= -b/a = 0/2 = 0$. This makes sense: $-2 + 2 = 0$. The symmetry about $x = 0$ means zeros are equal and opposite.

Q3. The product of the zeros of $p(x) = 2x^2 - 8$ is:

a) −4 b) 4

c) 8 d) −8

Solution: a) −4

Product = $c/a = -8/2 = -4$. Verify: $(-2) \times (2) = -4$

Q4. How many times does the graph of $p(x) = 2x^2 - 8$ intersect the x-axis?

a) 0 times
b) 1 time
c) 2 times
d) 3 times
Solution: c) 2 times
Since p(x) has two distinct real zeros ($x = -2$ and $x = 2$), its graph crosses the x-axis at exactly two distinct points.

Q5. The discriminant D of $p(x) = 2x^2 - 8$ is:
a) $D = 0$
b) $D < 0$
c) $D > 0$
d) $D = -8$
Solution: Here $a = 2$, $b = 0$, $c = -8$. $D = b^2 - 4ac = 0 - 4(2)(-8) = 0 + 64 = 64 > 0$. Since $D > 0$, there are two distinct real zeros confirmed by $x = \pm 2$.