Case Study Class 10 Maths Chapter 8 Introduction to Trigonometry

Case Study 5: Mountain Observation Activity

Two friends, Karan and Meena, are observing a mountain peak from the same side at different distances. Karan stands 500 metres from the base of the mountain and observes the peak at an angle of elevation of 45°. Meena stands 300 metres from the base and observes the peak at a steeper angle of 60°.

Questions:

(i) Using Karan's observation, find the height of the mountain.

(ii) Using Meena's observation, find the height of the mountain.

(iii) Are the two calculated heights consistent? What does this tell you?

(iv) Find the length of the line of sight from Karan to the peak.

(v) Meena walks 50 more metres towards the mountain. What is the new angle of elevation she would see? (Use tan values and round to nearest standard angle.)

Solution:

(i) Karan: $\tan 45° = h/500$, $1 = h/500$, $h = 500$ metres.

(ii) Meena: $\tan 60° = h/300$, $\sqrt{3} = h/300$, $h = 300\sqrt{3} \approx 519.6$ metres.

(iii) The two values (500 m and 519.6 m) are not exactly equal. This tells us that in a real scenario, slight differences arise from measurement error, the angle being approximate, or the base not being at the same level. In textbook problems, the scenario would be designed to give a unique height the inconsistency here shows the importance of precise angle measurement.

(iv) Karan's line of sight: $\sin 45° = 500/L$, $1/\sqrt{2} = 500/L$, $L = 500\sqrt{2} \approx 707$ metres.

(v) Meena's new distance = $300 - 50 = 250$ m. Using $h = 500$ m: $\tan \alpha = 500/250 = 2$, $\alpha \approx 63.4°$ (between 60° and 90°, close to 63°).

Case Study 6: Drone Flight Path Analysis

A drone takes off vertically from point A on the ground and rises to a height of 80 metres. It then flies horizontally to point B, which is 60 metres away from directly above A. A ground controller is standing at point C on the ground, directly below A. The controller wants to find the angle of elevation to the drone at B and the total distance of the drone from C.

Questions:

(i) Sketch the situation and identify the right-angled triangle.

(ii) Find the direct distance from C (controller) to the drone at B.

(iii) Find sin, cos, and tan of the angle of elevation from C to drone B.

(iv) Find the angle of elevation (use standard angle values or approximate).

(v) If the drone needs to return to C in a straight line, how far does it travel?

Solution:

(i) The triangle has: vertical height CA = 80 m (drone's altitude, the opposite side from angle at C), horizontal distance AB = 60 m (drone's horizontal flight, which projects to 60 m on the ground), and CB = hypotenuse (direct distance from controller to drone). The right angle is at the point directly below B on the ground (60 m from C along the ground).

(ii) Direct distance CB = $\sqrt{(80^2 + 60^2)} = \sqrt{(6400 + 3600)} = \sqrt{10000} = 100$ metres.

(iii) With θ = angle of elevation at C: $\sin\theta = 80/100 = 4/5 = 0.8$. $\cos\theta = 60/100 = 3/5 = 0.6$. $\tan\theta = 80/60 = 4/3 \approx 1.333$.

(iv) $\tan\theta = 4/3 \approx 1.333$. This is close to $\tan 53° \approx 1.327$. The angle of elevation is approximately 53°.

(v) The drone returns to C in a straight line: distance = CB = 100 metres (same as the direct distance calculated above).

Case Study 7: Sports and Angle Measurements

During a football match, a player at point P kicks the ball towards the goal. The goal crossbar is at a height of 2.44 metres. The player is standing 10 metres from the goal line. The goalkeeper is positioned at a distance of 4 metres from the goal line, directly in front of the goal. All three positions are in the same vertical plane.

Questions:

(i) Find the angle of elevation from the player at P to the top of the crossbar.

(ii) Find sin, cos, and tan of this angle.

(iii) The ball is kicked at an angle of 30° from the ground. At a horizontal distance of 10 m, what height does it reach? (Use $\tan 30° = 1/\sqrt{3}$.)

(iv) Does the ball clear the crossbar (height 2.44 m) if kicked at 30°?

(v) At what minimum angle must the ball be kicked to just clear the crossbar?

Solution:

(i) Player is 10 m from the goal. Crossbar height = 2.44 m. $\tan \theta = 2.44/10 = 0.244$. $\theta = \tan^{-1}(0.244) \approx 13.7°$ the player needs to kick at slightly above this angle.

(ii) With $\theta \approx 13.7°$, opposite = 2.44, adjacent = 10: Hypotenuse = $\sqrt{(2.44^2 + 10^2)} = \sqrt{(5.95 + 100)} = \sqrt{105.95} \approx 10.29$ m. $\sin \theta = 2.44/10.29 \approx 0.237$. $\cos \theta = 10/10.29 \approx 0.972$. $\tan \theta = 2.44/10 = 0.244$.

(iii) Ball kicked at 30°: height at 10 m horizontal distance. $\tan 30° = \text{height}/10$, $1/\sqrt{3} = h/10 \rightarrow h = 10/\sqrt{3} \approx 10/1.732 \approx 5.77$ metres.

(iv) Ball reaches 5.77 m at 10 m horizontal distance. Crossbar is at 2.44 m. Since 5.77 m > 2.44 m, the ball clears the crossbar comfortably when kicked at 30°.

(v) Minimum angle: $\tan \alpha = 2.44/10 = 0.244$. $\alpha = \tan^{-1}(0.244) \approx 13.7°$. Any angle greater than ≈ 13.7° will clear the crossbar at 10 m distance.