Case Study Questions on Chapter 10 Circles for Class 10 with Answers

Case Study Question 1: Tangent at the Point of Contact of Two Circles

Read the following situation carefully and answer the questions that follow:
Two circles with centres O and O' touch internally. The smaller circle has centre O' and radius 3 cm. The larger circle has centre O and radius 8 cm. They touch at point T. A tangent is drawn to the smaller circle at a point A on the smaller circle (A is different from T).
(i) How many common tangents do two internally touching circles have?
(ii) What is the distance OO'?
(iii) Is the common tangent at T also a tangent to the larger circle?
(iv) The tangent to the smaller circle at A is also a chord of the larger circle. True or False?
Answers:
(i) When two circles touch internally, they have exactly 1 common tangent, which is drawn at the point of contact T.
(ii) For internally touching circles:
$OO' = R - r = 8 - 3 = 5$ cm
(iii) Yes. The common internal tangent at T is tangent to both circles at the same point T.
(iv) True. The tangent to the smaller circle at A is not tangent to the larger circle. Since the smaller circle lies inside the larger circle, this tangent intersects the larger circle at two distinct points. Therefore, it is a chord of the larger circle.

Case Study Question 2: Bicycle Wheel and the Ground

Read the following situation carefully and answer the questions that follow:

Ramesh is riding a bicycle. The wheel of the bicycle has a radius of 35 cm. At a particular moment, the wheel touches the flat road at exactly one point. A straight line (the road) is in contact with the wheel at this one point. The centre of the wheel is at point O and the point of contact is A.
(i) What is the relationship between the road (tangent) and the line OA?
(ii) If a stone is stuck at point A on the wheel, and the distance from the stone to a nearby wall (measured along the ground from the base of the wall) is 120 cm, what is the distance from the centre O of the wheel to the base of the wall?
(iii) How many tangents can be drawn to this circle (wheel) from a point on the road that is outside the wheel?
(iv) Name the theorem that defines the relationship between OA and the tangent.
(v) If another tangent is drawn to the same wheel from the same external point on the road, how does its length compare to the first tangent?

Answers:

(i) The road (tangent) is perpendicular to OA (the radius at the point of contact). This is a direct application of Theorem 10.1: the tangent at any point of a circle is perpendicular to the radius through the point of contact.
(ii) Let B be the base of the wall on the ground. Then AB = 120 cm (along the ground), OA = 35 cm (radius, vertical to ground), and OB is the hypotenuse.
$OB^2 = OA^2 + AB^2$
$OB^2 = 35^2 + 120^2$
$OB^2 = 1225 + 14400 = 15625$
OB = 125 cm
(iii) From a point outside the circle, exactly 2 tangents can be drawn.
(iv) Theorem 10.1: The tangent at any point of a circle is perpendicular to the radius through the point of contact.
(v) By Theorem 10.2, both tangents from the same external point are equal in length. So the second tangent is also equal to the first.

Case Study Question 3: The Tangent from a Mobile Tower

Read the following situation carefully and answer the questions that follow:
A circular restricted zone around a mobile tower has centre O and radius 10 m. A road passes outside this zone. From a point P on the road, two straight boundary

ropes are tied to the edge of the restricted zone, each touching the boundary at exactly one point. The two points of contact are M and N. PM = 24 m.

(i) Find OP, the distance from P to the centre of the restricted zone.

(ii) Is PN = PM? Why?

(iii) Find the area of triangle OPM.

(iv) A third point Q lies on the line PM. If OQ=10 m, is Q on the boundary, inside, or outside the restricted zone?

(v) How many tangents can be drawn to the zone boundary from point P?

Answers:

(i) OM ⊥ PM (Theorem 10.1), so triangle OMP is right-angled at M.

$OP^2 = OM^2 + PM^2$

$OP^2 = 10^2 + 24^2$

$OP^2 = 100 + 576 = 676$

OP = 26 m

(ii) Yes, PN = PM = 24 m. By Theorem 10.2, the two tangents from an external point are equal in length.

(iii) Area of triangle OPM:

Base = PM = 24 m, Height = OM = 10 m

Area = ½ × 24 × 10 = 120 m²

(iv) OQ = 10 m = radius. Since OQ equals the radius, Q lies exactly on the boundary of the restricted zone.

(v) Exactly 2 tangents can be drawn from an external point P to the boundary of the circle.