Case Study for Class 10 Maths Chapter 11 Areas Related to Circles

Case Study 5: Circular Floor Decoration

An interior designer is creating a circular decorative pattern on the floor of a hotel lobby. The pattern consists of a large circle of radius 21 cm divided into 6 equal sectors, each painted a different colour. The designer wants to know the area of each coloured sector and the length of coloured border tape needed for the curved edge of each sector. (Use $\pi = 22/7$)

Questions:

(i) What is the central angle of each of the 6 equal sectors?

(ii) What is the total area of the large circle?

(iii) What is the area of one sector?

(iv) What is the arc length of one sector (the curved border)?

(v) What is the total length of border tape needed to outline the curved edges of all 6 sectors?

Solution:

Answer 1: The full circle is 360°, divided into 6 equal sectors.

Central angle of each sector = $360° \div 6 = 60°$

Answer 2: Radius, $r = 21$ cm

Area = $\pi r^2 = (22/7) \times 21 \times 21 = (22/7) \times 441 = 22 \times 63 = 1386$ cm²

Answer 3: Area of one sector = $(\theta/360°) \times \pi r^2 = (60/360) \times 1386 = (1/6) \times 1386 = 231$ cm²

Answer 4: Arc length of one sector = $(\theta/360°) \times 2\pi r$

$= (60/360) \times 2 \times (22/7) \times 21$

$= (1/6) \times 2 \times 22 \times 3$

$= (1/6) \times 132$

= 22 cm

Answer 5: Total tape needed = Arc length of one sector × number of sectors

$= 22 \times 6 = 132$ cm

This equals the full circumference of the circle, which makes sense since the 6 arc lengths together make up the complete circle boundary.

Case Study 6: Bicycle Wheel Measurements

Aman buys a new bicycle with wheels of diameter 70 cm. He wants to know how far the bicycle travels when the wheel completes a certain number of rotations, which will help him understand his cycling distance without using a tracking device. His father, an engineering graduate, helps him apply circle formulas to solve this problem. (Use $\pi = 22/7$)

Questions:

(i) What is the radius of the bicycle wheel?

(ii) What is the circumference of the wheel?

(iii) How far does the bicycle travel in one complete rotation of the wheel?

(iv) How far does the bicycle travel in 500 rotations of the wheel?

(v) How many rotations does the wheel make to cover a distance of 3.3 km?

Solution:

Answer 1: Diameter = 70 cm

Radius = $70 \div 2 = 35$ cm

Answer 2: Circumference = $2\pi r = 2 \times (22/7) \times 35 = 2 \times 22 \times 5 = 220$ cm

Answer 3: In one complete rotation, the wheel covers a distance equal to its circumference.

Distance per rotation = 220 cm (or 2.2 metres)

Answer 4: Distance in 500 rotations = 220 × 500 = 110,000 cm

Converting to metres: 110,000 ÷ 100 = 1100 metres (or 1.1 km)

Answer 5: Total distance = 3.3 km = 3,300 metres = 330,000 cm

Number of rotations = Total distance ÷ Circumference

= 330,000 ÷ 220 = 1500 rotations

Case Study 7: Sports Stadium Design

A sports stadium has a circular field used for athletics events. The field has a radius of 49 metres. For an upcoming event, officials need to mark a sector-shaped area of the field, with a central angle of 90°, for a special performance during the opening ceremony. They also want to calculate the boundary length of this sector for placing temporary barriers. (Use $\pi = 22/7$)

Questions:

(i) What is the total area of the circular field?

(ii) What is the area of the 90° sector marked for the performance?

(iii) What is the arc length of this 90° sector?

(iv) What is the total perimeter of the sector (including the two straight radius sides and the curved arc)?

(v) What fraction of the total field area does this sector represent?

Solution:

Answer 1: Radius, $r = 49$ m

Area = $\pi r^2 = (22/7) \times 49 \times 49 = (22/7) \times 2401 = 22 \times 343 = 7546$ m²

Answer 2: Area of 90° sector = $(\theta/360°) \times \pi r^2 = (90/360) \times 7546 = (1/4) \times 7546 = 1886.5$ m²

Answer 3: Arc length = $(\theta/360°) \times 2\pi r$

= (90/360) × 2 × (22/7) × 49

= (1/4) × 2 × 22 × 7

= (1/4) × 308

= 77 m

Answer 4: Perimeter of sector = Arc length + 2 × radius

= 77 + 2 × 49

= 77 + 98

= 175 m

Answer 5: Fraction of total area = Sector area ÷ Total area = 1886.5 ÷ 7546 = 1/4

Since the sector angle is 90° out of 360°, this confirms the sector represents exactly one quarter of the field.