

Case Study for Class 10 Maths Chapter 12 Surface Areas and Volumes

Case Study 5: Designing a Hemispherical Fountain

A city park is installing a hemispherical decorative water fountain bowl. The bowl has a radius of 21 cm. Water is pumped into the bowl until it is completely full, and then it overflows decoratively over the edges. The park designer wants to calculate the amount of water the bowl can hold and the surface area of material needed to construct the curved bowl. (Use $\pi = 22/7$)

Questions:

- (i) What is the volume of water the hemispherical bowl can hold?
- (ii) What is the curved surface area of the bowl (the inner curved surface in contact with water)?
- (iii) What is the total surface area of the bowl if it also has a flat circular rim at the top?
- (iv) How many litres of water does the bowl hold when full?
- (v) If the bowl is to be coated with a waterproof layer costing ₹15 per cm^2 , what is the total coating cost for the curved surface?

Solution:

Answer 1: Radius, $r = 21$ cm

$$\text{Volume} = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$$

$$= \frac{2}{3} \times \frac{22}{7} \times 9261$$

$$= \frac{2}{3} \times 29,106$$

$$= 19,404 \text{ cm}^3$$

Answer 2: Curved Surface Area = $2\pi r^2 = 2 \times \frac{22}{7} \times 21 \times 21$

$$= 2 \times \frac{22}{7} \times 441$$

$$= 2 \times 1386$$

$$= 2772 \text{ cm}^2$$

Answer 3: Total Surface Area = $3\pi r^2 = 3 \times (22/7) \times 441$

$$= 3 \times 1386$$

$$= 4158 \text{ cm}^2$$

Answer 4: Since $1000 \text{ cm}^3 = 1 \text{ litre}$:

$$\text{Capacity} = 19,404 \div 1000 = 19.404 \text{ litres}$$

Answer 5: Coating cost = Curved Surface Area \times rate

$$= 2772 \times 15$$

$$= ₹41,580$$

Case Study 6: Metal Sphere Production

A metal manufacturing company produces small spherical ball bearings. Each spherical ball bearing has a radius of 3 cm. The factory manager wants to melt down a batch of these spherical ball bearings to recast them into a single large solid cylinder of radius 6 cm. He asks the production team to calculate how many spherical balls are needed and what height the resulting cylinder will have. (Use $\pi = 22/7$)

Questions:

- (i) What is the volume of one spherical ball bearing?
- (ii) If 224 such spherical balls are melted together, what is the total volume of metal obtained?
- (iii) If this metal is recast into a cylinder of radius 6 cm, what is the height of the resulting cylinder?
- (iv) What is the surface area of one original spherical ball bearing?
- (v) Verify that the volume of the cylinder equals the total volume of the 224 spheres.

Solution:

Answer 1: Radius, $r = 3$ cm

$$\text{Volume of one sphere} = \left(\frac{4}{3}\right)\pi r^3 = \left(\frac{4}{3}\right) \times \left(\frac{22}{7}\right) \times 3 \times 3 \times 3$$

$$= \left(\frac{4}{3}\right) \times \left(\frac{22}{7}\right) \times 27$$

$$= \left(\frac{4}{3}\right) \times 84.857$$

$$= 113.14 \text{ cm}^3 \text{ (approximately)}$$

$$\text{For cleaner calculation: } \left(\frac{4}{3}\right) \times \left(\frac{22}{7}\right) \times 27 = \frac{4 \times 22 \times 27}{3 \times 7} = \frac{2376}{21} = 113.14 \text{ cm}^3$$

Answer 2: Total volume = Volume of one sphere \times number of spheres

$$= 113.14 \times 224$$

$$= 25,344 \text{ cm}^3 \text{ (approximately)}$$

Answer 3: Volume of cylinder = $\pi R^2 H$

$$25,344 = \left(\frac{22}{7}\right) \times 6 \times 6 \times H$$

$$25,344 = \left(\frac{22}{7}\right) \times 36 \times H$$

$$25,344 = 113.14 \times H$$

$$H = 25,344 \div 113.14$$

$$H \approx 224 \text{ cm}$$

Answer 4: Surface Area of one sphere = $4\pi r^2 = 4 \times \left(\frac{22}{7}\right) \times 3 \times 3$

$$= 4 \times \left(\frac{22}{7}\right) \times 9$$

$$= 4 \times 28.29$$

$$= 113.14 \text{ cm}^2$$

Answer 5: Volume of cylinder = $\pi R^2 H = \left(\frac{22}{7}\right) \times 36 \times 224$

$$= \left(\frac{22}{7}\right) \times 8064$$

$$= 22 \times 1152$$

$$= 25,344 \text{ cm}^3$$

This matches the total volume of the 224 spheres calculated in Answer 2, confirming that volume is conserved during the melting and recasting process.

Case Study 7: Converting a Solid Shape into Another Shape

A jewellery workshop has a solid metal cone with a radius of 6 cm and a height of 24 cm. The goldsmith wants to melt this cone completely and recast it into a number of small solid spheres, each with a radius of 3 cm, to make decorative beads. He asks his apprentice to determine how many beads can be made from the melted cone. (Use $\pi = 22/7$)

Questions:

- (i) What is the volume of the original solid cone?
- (ii) What is the volume of one small spherical bead?
- (iii) How many spherical beads can be made from the melted cone?
- (iv) If instead the cone is recast into a single cylinder with the same radius as the cone's base (6 cm), what would be the height of the new cylinder?
- (v) Why does the height of the cylinder formed in Question 4 differ from the height of the original cone?

Solution:

Answer 1: Radius, $r = 6$ cm, Height, $h = 24$ cm

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \left(\frac{22}{7}\right) \times 6 \times 6 \times 24$$

$$= \frac{1}{3} \times \left(\frac{22}{7}\right) \times 36 \times 24$$

$$= \frac{1}{3} \times \left(\frac{22}{7}\right) \times 864$$

$$= \frac{1}{3} \times 2716.57$$

$$= 905.14 \text{ cm}^3 \text{ (approximately, using } 22/7)$$

$$\text{More precisely: } \frac{1}{3} \times \left(\frac{22}{7}\right) \times 864 = \frac{22 \times 864}{3 \times 7} = \frac{19008}{21} = 905.14 \text{ cm}^3$$

Answer 2: Radius of bead, $r = 3$ cm

$$\text{Volume of one sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 3 \times 3 \times 3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 27$$

$$= 113.14 \text{ cm}^3$$

Answer 3: Number of beads = Volume of cone \div Volume of one bead

$$= 905.14 \div 113.14$$

= 8 beads (approximately)

Answer 4: Since volume remains constant during conversion:

$$\text{Volume of cylinder} = \text{Volume of cone} = 905.14 \text{ cm}^3$$

$$\pi R^2 H = 905.14$$

$$\frac{22}{7} \times 6 \times 6 \times H = 905.14$$

$$\frac{22}{7} \times 36 \times H = 905.14$$

$$113.14 \times H = 905.14$$

$$H = 905.14 \div 113.14$$

$$H \approx 8 \text{ cm}$$

Answer 5: The original cone has a height of 24 cm, but the new cylinder (with the same radius) has a height of only 8 cm. This is because a cone occupies only one-third of the volume of a cylinder with the same base radius and height (Volume of cone = $\frac{1}{3}$ \times Volume of cylinder of same r and h). When the cone's material is packed into a cylinder of the same radius, it only needs one-third of the original height to hold the same volume, which is why $24 \div 3 = 8$ cm. This demonstrates the direct mathematical relationship between cone and cylinder volumes when they share the same base radius.