Case Study for Class 9 Maths Chapter 3 The World of Numbers

Case Study 1: Operations on Real Numbers

Meena writes four statements on the board about arithmetic with rational and irrational numbers. She challenges her classmates to verify each statement using the properties of real numbers.

The four statements:

1. $\sqrt{3} + \sqrt{3} = 2\sqrt{3}$
2. $\sqrt{2} \times \sqrt{8} = 4$
3. $(\sqrt{5} + 1)(\sqrt{5} - 1) = 4$
4. $\sqrt{2} + (-\sqrt{2}) = 0$

Questions:

(i) Is statement 1 correct? What type of number is $2\sqrt{3}$?

(ii) Verify statement 2. Is the result rational or irrational?

(iii) Verify statement 3 using the identity $(a + b)(a - b) = a^2 - b^2$.

(iv) What does statement 4 show about irrational numbers?

(v) What can you conclude: is the sum of two irrationals always irrational?

Solution:

(i) $\sqrt{3} + \sqrt{3} = 2\sqrt{3}$. This is correct it follows the same rule as $x + x = 2x$. Since $\sqrt{3}$ is irrational and 2 is rational, $2\sqrt{3}$ is irrational.

(ii) $\sqrt{2} \times \sqrt{8} = \sqrt{(2 \times 8)} = \sqrt{16} = 4$. The result is 4, which is rational. This shows that the product of two irrational numbers can sometimes be rational.

(iii) $(\sqrt{5} + 1)(\sqrt{5} - 1) = (\sqrt{5})^2 - 1^2 = 5 - 1 = 4$. The identity confirms the result. The product is rational.

(iv) Statement 4 shows that $\sqrt{2} + (-\sqrt{2}) = 0$. Every irrational number has an additive inverse (its negative), and their sum is zero a rational number. So the sum of two irrationals can be rational.

(v) No. The sum of two irrational numbers is not always irrational. Counter-example: $\sqrt{3} + (-\sqrt{3}) = 0$ (rational). However, $\sqrt{2} + \sqrt{3}$ is irrational. The result depends on the specific numbers.

Case Study 2: Comparing Different Types of Numbers

Questions

(i) Which of the six numbers are integers?

(ii) Write all numbers that are rational.

(iii) Which numbers are irrational? Justify.

(iv) Is every integer also a rational number? Give a reason.

(v) Arrange the six numbers in ascending order on the number line.

Solution:

(i) −3 and 0 are integers (whole numbers and their negatives, without fractions).

(ii) −3, 0, 5/7, and 4.5 are all rational. They can each be written as p/q: −3 = −3/1, 0 = 0/1, 5/7 = 5/7, 4.5 = 9/2.

(iii) $\sqrt{2}$ and π are irrational. $\sqrt{2}$ = 1.41421356… (never repeats) and π = 3.14159265… (never repeats). Neither can be written as p/q.

(iv) Yes. Every integer n can be written as n/1, which is in the form p/q with $q = 1 \neq 0$. So every integer satisfies the definition of a rational number.

(v) Ascending order: $-3 < 0 < 5/7$ (≈ 0.71) $< \sqrt{2}$ (≈ 1.41) $< 4.5 < \pi$ (≈ 3.14). Wait $4.5 > \pi$ (3.14), so the correct order is: $-3 < 0 < 5/7 < \sqrt{2} < \pi < 4.5$.

Case Study 3: Number Patterns and Real Numbers

Deepa observes the following pattern in her notebook: 1/1, 1/2, 1/3, 1/4, 1/5… She also notices a second pattern: √1, √2, √3, √4, √5… She wants to know which terms in each pattern are rational and which are irrational.

Questions

(i) Are all terms in the first pattern (1/n) rational?

(ii) In the second pattern, which terms are rational?

(iii) In the second pattern, which terms are irrational?

(iv) Is √9 rational? Justify.

(v) Can any term in the pattern 1/n be irrational? Explain why or why not.

Solution:

(i) Yes. Every term 1/n is of the form p/q where p = 1 and q = n (a positive integer). So all terms 1/1, 1/2, 1/3… are rational numbers.

(ii) In the pattern √1, √2, √3, √4, √5…, the rational terms are those where the number under the root is a perfect square: √1 = 1, √4 = 2, √9 = 3, √16 = 4, and so on.

(iii) √2, √3, √5, √6, √7, √8, √10… are all irrational because they are square roots of non-perfect squares. Their decimal expansions never terminate and never repeat.

(iv) Yes. √9 = 3, and 3 = 3/1 satisfies the p/q definition with integers p = 3 and q = 1. So √9 is rational.

(v) No. Every term 1/n involves only integers in the numerator (1) and denominator (n). Since both p and q are integers and n ≠ 0, every term 1/n is rational by definition. The pattern cannot produce irrational values.