Case Study on Chapter 4: Exploring Algebraic Identities for Class 9

Case Study1: The Framer's Mount Board

A picture framer designs a rectangular mount board so that its area, in square centimetres, is given by the expression $x^2 + 9x + 20$, where x is a measurement used in his design template.
(i) Find two numbers a and b such that $a + b = 9$ and $ab = 20$.
(ii) Use these values to factorise $x^2 + 9x + 20$ and write down expressions for the length and breadth of the board.
(iii) If x = 10 cm, find the actual length and breadth of the mount board.
(iv) Find the area of the board when x = 10 cm, and confirm it matches $x^2 + 9x + 20$ at $x = 10$.
OR
Write down the general identity that connects $(x + a)(x + b)$ to $x^2 + 9x + 20$.
Solution:
(i) $a = 4$ and $b = 5$, since $4 + 5 = 9$ and $4 \times 5 = 20$.

(ii) $x^2 + 9x + 20 = (x + 4)(x + 5)$, so the board's length and breadth are $(x + 5)$ and $(x + 4)$ units.

(iii) At $x = 10$: length = $10 + 5 = 15$ cm and breadth = $10 + 4 = 14$ cm.

(iv) Area = $15 \times 14 = 210$ cm². Checking directly: $10^2 + 9(10) + 20 = 100 + 90 + 20 = 210$ cm², which matches.

OR: $(x + a)(x + b) = x^2 + (a + b)x + ab$.

Case Study 2: The Trader's Cost Formula

A trader is reviewing a packaging cost formula before showing it to his accountant. After some calculation, the relevant relationship simplifies to the rational expression $(x^2 - 2x - 15) / (4x^2 - 100)$.
(i) Factorise the numerator, $x^2 - 2x - 15$.

(ii) Factorise the denominator, $4x^2 - 100$, using a suitable identity.
(iii) Simplify the rational expression fully, assuming the denominator is not zero.
(iv) Evaluate the simplified expression at $x = 7$.
OR
Why must we assume the denominator is not equal to zero while simplifying this expression?
Solution:
(i) We need $a + b = -2$ and $ab = -15$, which gives $a = -5$ and $b = 3$. So $x^2 - 2x - 15 = (x - 5)(x + 3)$.

(ii) $4x^2 - 100 = 4(x^2 - 25) = 4(x - 5)(x + 5)$, using the identity $x^2 - y^2 = (x - y)(x + y)$ with $y = 5$.

(iii) $(x - 5)(x + 3) / [4(x - 5)(x + 5)] = (x + 3) / [4(x + 5)]$, after cancelling the common factor $(x - 5)$.

(iv) At $x = 7$: $(7 + 3) / [4(7 + 5)] = 10 / 48 = 5/24$.

OR: Dividing by zero is undefined. The denominator $4x^2 - 100$ becomes zero at $x = 5$ or $x = -5$, so the original expression simply doesn't exist at those values. Simplified form is only valid everywhere except there.

Case Study 3: Three Friends' Community Fridge Fund

Three friends pooled money for a community fridge project. Together, their contributions add up to 15 (in hundreds of rupees), the sum of the squares of their individual contributions is 83, and the product of their contributions is 80.
(i) Using the identity $(x+y+z)^2 = x^2+y^2+z^2+2(xy+yz+zx)$, find the value of $xy + yz + zx$.
(ii) Using the identity $x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$, find the value of $x^3+y^3+z^3-3xyz$.
(iii) Hence find the value of $x^3+y^3+z^3$.
(iv) If instead the three contributions had added up to exactly 0, what would $x^3+y^3+z^3$ equal?
OR

State the special identity that applies whenever $x + y + z = 0$.

Solution:

(i) $15^2 = 83 + 2(xy+yz+zx) \Rightarrow 225 - 83 = 2(xy+yz+zx) \Rightarrow xy+yz+zx = 71$.

(ii) $x^3+y^3+z^3-3xyz = 15 \times (83 - 71) = 15 \times 12 = 180$.

(iii) $x^3+y^3+z^3 = 180 + 3(80) = 180 + 240 = 420$.

(iv) If $x+y+z = 0$, the right-hand side of the identity becomes 0, so $x^3+y^3+z^3-3xyz = 0$, which means $x^3+y^3+z^3 = 3xyz$.

OR: If $x+y+z = 0$, then $x^3+y^3+z^3 = 3xyz$.