Case Study on Class 10 Maths Chapter 4 'Quadratic Equations'

Case Study 5: Construction and Architecture Planning

An architect is designing a rectangular conference hall for a new office building. The floor area of the hall must be exactly 300 square metres. The architect wants the length of the hall to be 5 metres more than its breadth so that the hall is wide enough for comfortable movement. A structural engineer reviews the plan and asks the junior team members who are recently graduated students to verify the dimensions by solving the appropriate quadratic equation.

Questions:

(i) Write a quadratic equation to represent the dimensions of the hall.

(ii) Solve the equation using the quadratic formula.

(iii) What are the exact dimensions of the hall?

(iv) What is the discriminant of the equation? What does it tell you?

(v) If the height of the hall is 4 metres, what is the total floor-to-ceiling surface area of the four walls?

Solution:

Answer 1: Let breadth = x metres. Length = x + 5 metres. Area = 300 m^2.

$x(x + 5) = 300$ $x^2 + 5x - 300 = 0$

Answer 2: $a = 1$, $b = 5$, $c = -300$

$D = 5^2 - 4(1)(-300) = 25 + 1200 = 1225$

$\sqrt{1225} = 35$

$x = (-5 \pm 35) / 2$

Taking positive root: $x = (-5 + 35) / 2 = 30 / 2 = 15$

Answer 3: Breadth = 15 metres Length = 15 + 5 = 20 metres

Verification: $15 \times 20 = 300$ m²

Answer 4: D = 1225, which is greater than zero.

Since D > 0, the equation has two distinct real roots: x = 15 and x = −20. Since a negative breadth makes no physical sense, only x = 15 is accepted.

Answer 5: The hall has two pairs of walls: Two walls of length 20 m × height 4 m each: area = $2 \times 20 \times 4 = 160$ m² Two walls of breadth 15 m × height 4 m each: area = $2 \times 15 \times 4 = 120$ m²

Total wall surface area = $160 + 120 = 280$ m²

Case Study 6: Pathway and Flooring Design

A square park of side 10 metres is surrounded by a pathway of uniform width on all four sides. The total area of the park along with the pathway is 196 square metres. A landscaping company is hired to lay tiles on the pathway only (not on the grass of the park). The company manager asks a Class 10 student doing an internship to calculate the width of the pathway using quadratic equations.

Questions:

(i) If the width of the pathway is x metres, what is the total side length of the outer boundary (park plus pathway)?

(ii) Form a quadratic equation using the total area given.

(iii) Solve the equation to find the width of the pathway.

(iv) What is the area of the pathway alone (excluding the park)?

(v) If tiles cost ₹200 per square metre, what is the total cost of tiling the pathway?

Solution:

Answer 1: The park is 10 m × 10 m. The pathway adds x metres on each side left, right, top, and bottom.

Total side length of the outer boundary = $10 + x + x = 10 + 2x$ metres.

Answer 2: Total area = $(10 + 2x)^2 = 196$

$(10 + 2x)^2 = 196$

Taking the positive square root first: $10 + 2x = 14$ $2x = 4$ $x = 2$

Let us also write it as a quadratic: $(10 + 2x)^2 = 196$ $100 + 40x + 4x^2 = 196$ $4x^2 + 40x + 100 - 196 = 0$ $4x^2 + 40x - 96 = 0$ $x^2 + 10x - 24 = 0$

Answer 3: Solving $x^2 + 10x - 24 = 0$:

Find two numbers with product −24 and sum 10. 12 and −2: $12 \times (-2) = -24$ and $12 + (-2) = 10$

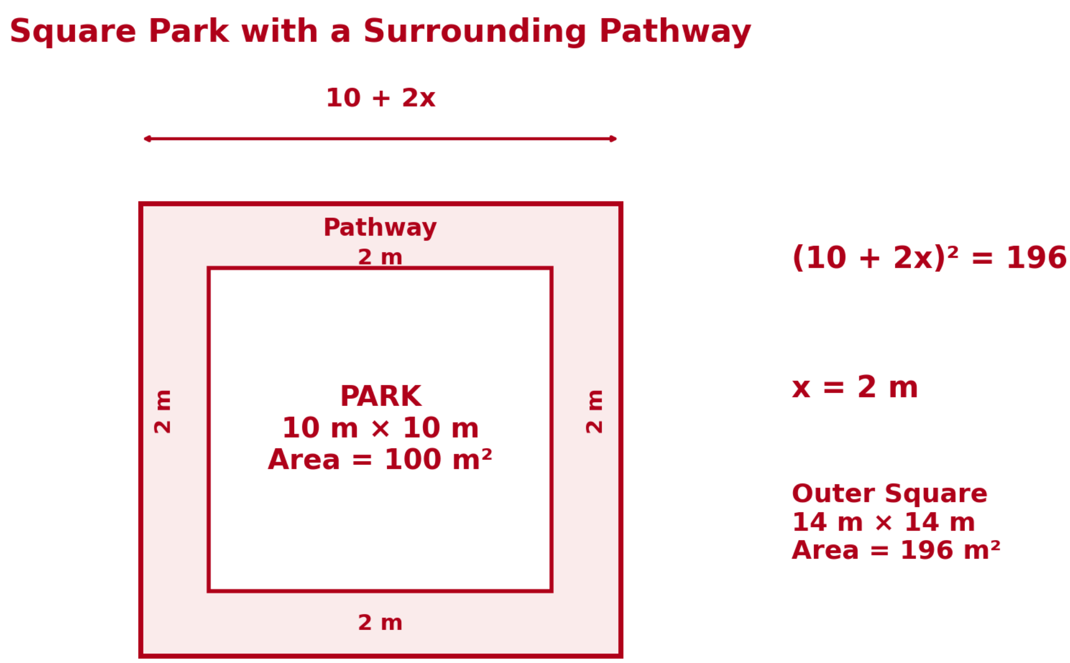
$(x + 12)(x - 2) = 0$ $x = -12$ or $x = 2$

Since width cannot be negative, x = 2 metres.

The pathway is 2 metres wide.

Answer 4: Total area (park + pathway) = 196 m² Area of park alone = 10 × 10 = 100 m² Area of pathway = 196 − 100 = 96 m²

Answer 5: Cost of tiling = 96 × 200 = ₹19,200



Case Study 7: Speed, Distance and Time Relationships

Riya and her father plan a road trip from City A to City B, a distance of 300 kilometres. On the return journey, Riya's father increases the average speed by 10 kilometres per hour, which reduces the journey time by 1 hour compared to the onward trip. Riya's uncle, a mathematics teacher, asks Riya to set up and solve a quadratic equation to find the speed during the onward journey.

Questions:

(i) If the speed during the onward journey is x km/h, write an expression for the time taken for the onward journey.

(ii) Write an expression for the time taken for the return journey.

(iii) Form a quadratic equation based on the difference in times.

(iv) Solve the equation to find the speed of the onward journey.

(v) What are the speeds and times for both the onward and return journeys?

Solution:

Answer 1: Distance = 300 km. Speed = x km/h.

Using Time = Distance ÷ Speed:

Time for onward journey = 300/x hours

Answer 2: Return speed = x + 10 km/h.

Time for return journey = 300/(x + 10) hours

Answer 3: The return journey is 1 hour shorter:

$300/x - 300/(x + 10) = 1$

Multiply throughout by x(x + 10):

$300(x + 10) - 300x = x(x + 10)$

$300x + 3000 - 300x = x^2 + 10x$

$3000 = x^2 + 10x$

$x^2 + 10x - 3000 = 0$

Answer 4: Solving $x^2 + 10x - 3000 = 0$ using the quadratic formula: $a = 1$, $b = 10$, $c = -3000$

$D = 100 - 4(-3000) = 100 + 12000 = 12100$

$\sqrt{12100} = 110$

$x = (-10 \pm 110) / 2$

Taking positive root: $x = (-10 + 110) / 2 = 100 / 2 = 50$ km/h

Answer 5: Onward speed = 50 km/h Onward time = 300 / 50 = 6 hours

Return speed = 50 + 10 = 60 km/h Return time = 300 / 60 = 5 hours

Difference in time = 6 − 5 = 1 hour

The verification confirms the answer is correct.