

Chapter 10 :Circles Notes for Class 10

Key Terms and Definitions

- Circle:

A circle is the set of all points in a plane that are at a fixed distance from a fixed point called the centre.

Centre: O, Radius: r

- Radius

The fixed distance from the centre of the circle to any point on it. All radii of a circle are equal in length.

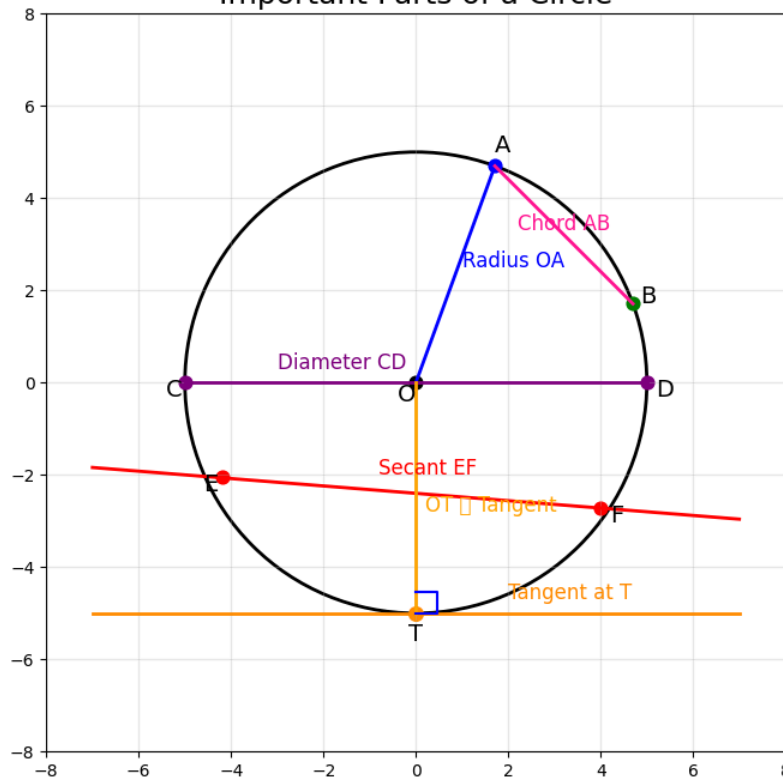
$$OA = OB = r$$

- Chord

A line segment whose both endpoints lie on the circle. The longest chord of a circle is the diameter.

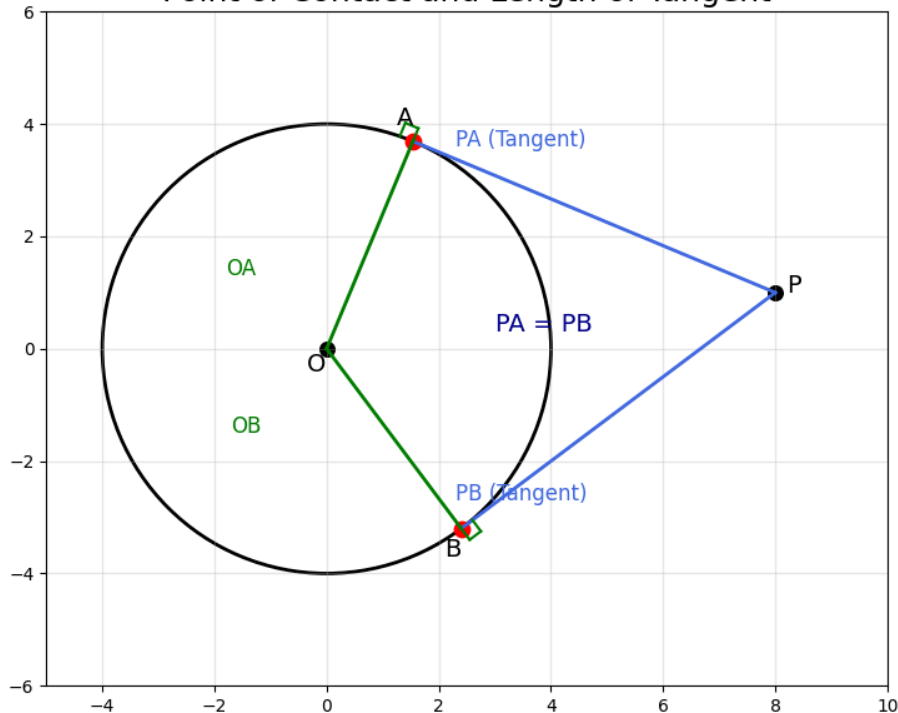
AB is a chord

Important Parts of a Circle



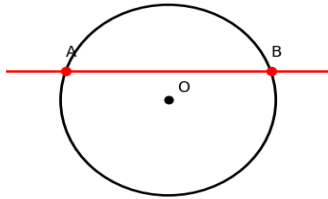
- **Diameter**
A chord that passes through the centre of the circle. Diameter = $2 \times$ Radius.
 $d = 2r$
- **Secant**
A line that intersects a circle at two distinct points. It cuts through the circle, creating a chord inside it.
2 common points
- **Tangent**
A line that touches a circle at exactly one point. That single point is called the point of contact (or point of tangency).
1 common point
- **Point of Contact**
The single point where the tangent meets the circle. The radius drawn to this point is perpendicular to the tangent.
P (tangent point)
- **Length of Tangent**
The distance from an external point to the point of contact. If P is external and PA is tangent, then PA is the length of the tangent from P.
 $PA = PB$ (external)

Point of Contact and Length of Tangent

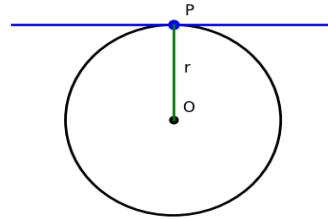


A Line and a Circle: Three Possibilities

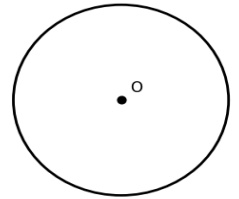
Secant
(2 Common Points)



A Line and a Circle — Three Possibilities
Tangent
(1 Common Point)

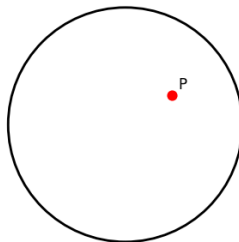


No Intersection
(0 Common Points)

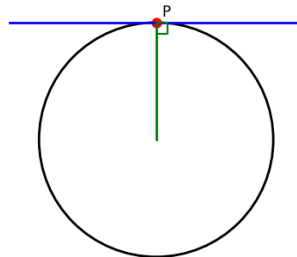


Number of Tangents from a Point to a Circle

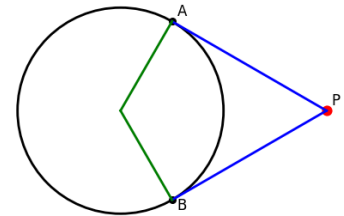
Inside the Circle
0 Tangents



Number of Tangents from a Point to a Circle
On the Circle
1 Tangent



Outside the Circle
2 Tangents



Position of Point	Number of Tangents	Reason
Inside the circle	0 (None)	Any line through this point will always cut the circle at two points, making it a secant and not a tangent.
On the circle	1 (Exactly one)	Only one tangent can be drawn at a point lying on the circle itself.

Outside the circle	2 (Exactly two)	Two tangents can always be drawn from an external point to the circle.
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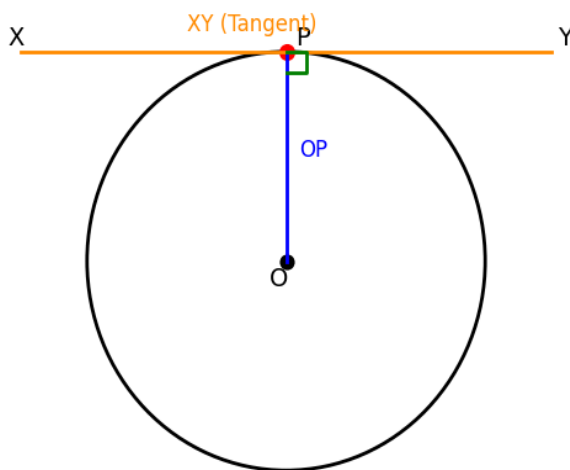
The Two Key Theorems

Theorem 10.1

The tangent at any point of a circle is perpendicular to the radius through the point of contact.

In simple words: If a line touches a circle at point P, then the radius OP is perfectly perpendicular (at 90°) to that tangent line at P.

Proof :



Given: Circle with centre O. XY is a tangent at point P.

To Prove: $OP \perp XY$

Take any point Q on the tangent XY, other than P, and join OQ.

Since XY is a tangent and Q is a different point on it, Q must lie outside the circle. (If Q were inside the circle, XY would become a secant, not a tangent.)

Since Q is outside the circle and P is on the circle:

$OQ > OP$ (radius)

This is true for every point Q on XY except P. So OP is the shortest distance from O to the line XY.

The shortest distance from a point to a line is always perpendicular to the line.

Therefore:

$OP \perp XY$
 Hence Proof

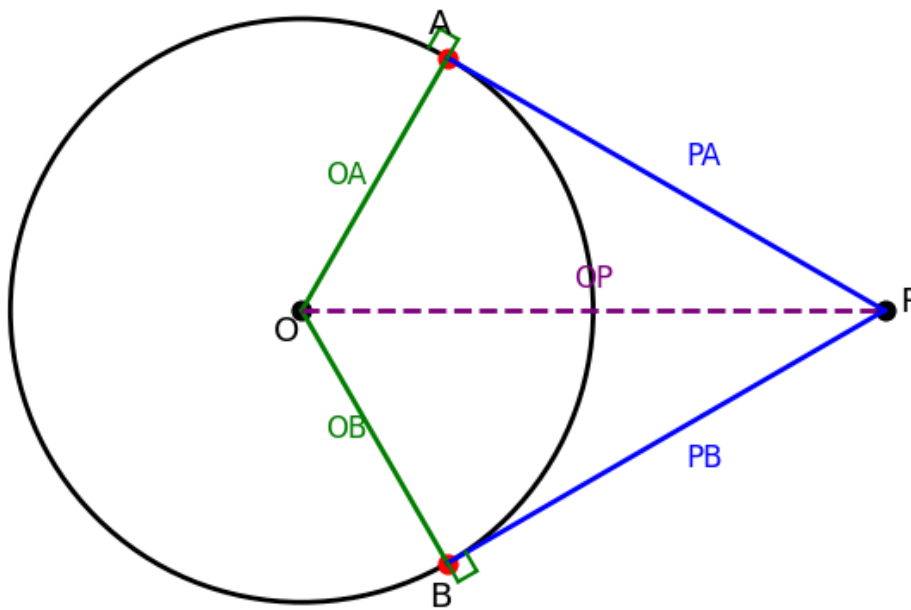
The converse is also true: A line drawn through the end of a radius and perpendicular to it is a tangent to the circle.

Theorem 10.2

The lengths of tangents drawn from an external point to a circle are equal.

In simple words: If you stand outside a circle and draw two tangent lines to it, the distance from your position to each tangent's touching point is the same. Always.

Proof:



Given: Circle with centre O. P is an external point. PA and PB are tangents touching the circle at A and B respectively.

To Prove: $PA = PB$

Join OA, OB, and OP.

By Theorem 10.1, the tangent is perpendicular to the radius at the point of contact:

$$\angle OAP = 90^\circ \text{ and } \angle OBP = 90^\circ$$

In triangles OAP and OBP, consider the following:

$$OA = OB \quad (\text{radii of same circle})$$

$$OP = OP \quad (\text{common side})$$

$$\angle OAP = \angle OBP = 90^\circ \text{ (from step 2)}$$

By the RHS (Right angle–Hypotenuse–Side) congruence rule:

$$\triangle OAP \cong \triangle OBP$$

By CPCT (Corresponding Parts of Congruent Triangles):

$$PA = PB$$

Hence Proof

Note:

$\angle OPA = \angle OPB$, which means OP bisects angle APB. Also, $\angle AOB + \angle APB = 180^\circ$ (supplementary).

Important Properties at a Glance

- The tangent at any point is perpendicular to the radius at that point: $OP \perp$ tangent.
- Tangents from an external point are equal in length: $PA = PB$.
- The tangents from an external point are equally inclined to the line joining the point to the centre: $\angle OPA = \angle OPB$
- $\angle AOB + \angle APB = 180^\circ$ (the angle subtended at centre and the angle at external point are supplementary)
- Tangents drawn at the ends of a diameter are parallel to each other.
- In a chord of a larger circle that is tangent to a smaller concentric circle, the chord is bisected at the point of contact.
- A parallelogram circumscribing a circle is always a rhombus.
- The perpendicular from the centre to a chord bisects it.

Solved Questions

Q1: A point P is 10 cm away from the centre of a circle. The radius of the circle is 6 cm. Find the length of the tangent drawn from P to the circle.

Solution: Given: $OP = 10 \text{ cm}$ | Radius $OT = 6 \text{ cm}$ | Find: PT

By Theorem 10.1, $OT \perp PT$, so triangle OTP is right-angled at T.

Applying Pythagoras' theorem in $\triangle OTP$:

$$OP^2 = OT^2 + PT^2$$

Substituting values:

$$10^2 = 6^2 + PT^2$$

$$100 = 36 + PT^2$$

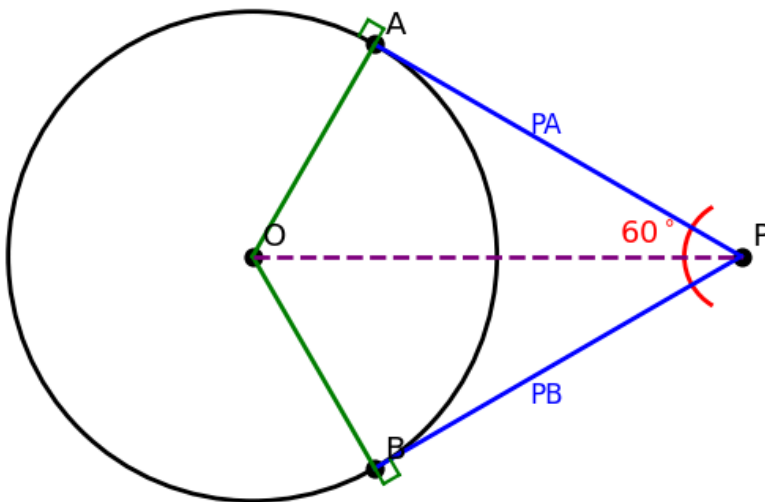
$$PT^2 = 64$$

$$\text{Therefore: } PT = \sqrt{64} = 8 \text{ cm}$$

Length of tangent $PT = 8 \text{ cm}$

Q2: Two tangents PA and PB are drawn from an external point P to a circle with centre O such that $\angle APB = 60^\circ$. Find $\angle OAB$.

Solution: Given: PA and PB are tangents from external point P. $\angle APB = 60^\circ$. Find $\angle OAB$.



By Theorem 10.2, $PA = PB$. So $\triangle PAB$ is isosceles. Therefore $\angle PAB = \angle PBA$.

In $\triangle PAB$: $\angle PAB + \angle PBA + \angle APB = 180^\circ$

$$\Rightarrow 2\angle PAB + 60^\circ = 180^\circ$$

$$\Rightarrow 2\angle PAB = 120^\circ$$

$$\Rightarrow \angle PAB = 60^\circ$$

By Theorem 10.1, $\angle OAP = 90^\circ$ (radius $OA \perp$ tangent PA at A).

Therefore:

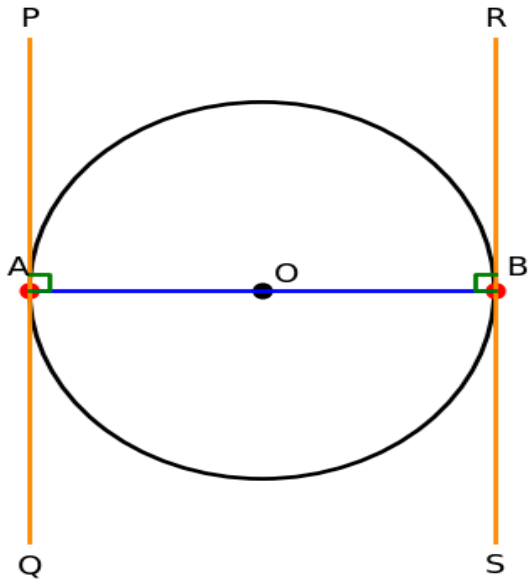
$$\angle OAB = \angle OAP - \angle PAB = 90^\circ - 60^\circ = 30^\circ$$

$$\Rightarrow \angle OAB = 30^\circ$$

Q3: Prove that the tangents drawn at the ends of a diameter of a circle are parallel to each other.

Solution: Given: Circle with centre O and diameter AB. PQ is tangent at A, and RS is tangent at B.

Prove: $PQ \parallel RS$.



By Theorem 10.1, the tangent at any point is perpendicular to the radius at that point.

At point A: $OA \perp PQ$, so $\angle OAP = 90^\circ$ (i.e., $\angle PAB = 90^\circ$)

At point B: $OB \perp RS$, so $\angle OBR = 90^\circ$ (i.e., $\angle RBA = 90^\circ$)

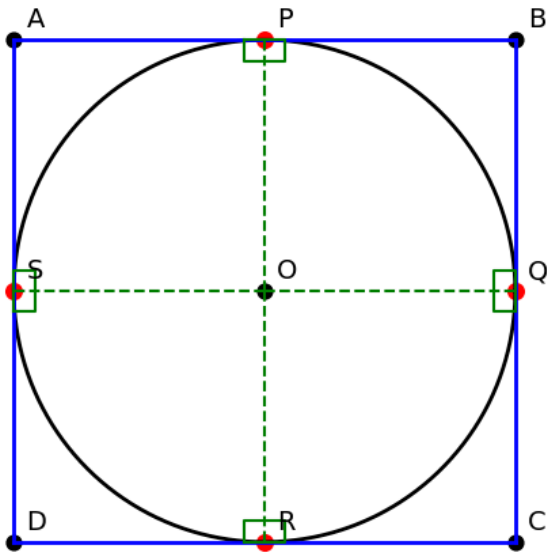
$\angle PAB = \angle RBA = 90^\circ$ (alternate interior angles formed by the transversal AB with lines PQ and RS)

Since alternate interior angles are equal, $PQ \parallel RS$.

The two tangents at the ends of the diameter are parallel.

Proved.

Q4: A quadrilateral ABCD is drawn to circumscribe a circle. Prove that $AB + CD = AD + BC$.



Solution: Given: Quadrilateral ABCD circumscribes a circle (i.e., all four sides are tangents to the circle). The circle touches the sides AB, BC, CD, DA at points P, Q, R, S respectively.

By Theorem 10.2, tangents from an external point to a circle are equal.

From vertex A (external point): $AP = AS$

From vertex B: $BP = BQ$

From vertex C: $CQ = CR$

From vertex D: $DR = DS$

Adding all four equations:

$$(AP + BP) + (CQ + DR) = (AS + DS) + (BQ + CR)$$

$$AB + CD = AD + BC$$

$$AB + CD = AD + BC$$

