

Chapter 12: Surface Areas and Volumes Notes for Class 10

Basic Solid Formulas

- **CSA (Curved Surface Area):** Area of only the curved/lateral surface, excludes flat bases.
- **LSA (Lateral Surface Area):** Same as CSA. Used especially for cubes and cuboids.
- **TSA (Total Surface Area):** CSA + area of all flat bases. The entire outer area.
- **Volume:** The amount of 3D space a solid occupies (cubic units).
- **Slant height (l):** The distance from the apex of a cone to any point on the base circle's edge (not the perpendicular height).

Formula Table: All Shapes

Shape	CSA / LSA	Total Surface Area (TSA)	Volume	Key Note
Cube (edge = a)	$4a^2$	$6a^2$	a^3	Diagonal = $a\sqrt{3}$
Cuboid (l, b, h)	$2h(l+b)$	$2(lb+bh+hl)$	lbh	Diagonal = $\sqrt{l^2+b^2+h^2}$
Cylinder (r, h)	$2\pi rh$	$2\pi r(r+h)$	$\pi r^2 h$	TSA = CSA + 2 circular bases
Cone (r, h, l)	πrl	$\pi r(r+l)$	$\frac{1}{3}\pi r^2 h$	$l = \sqrt{r^2 + h^2}$
Sphere (r)	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$	TSA = CSA (no flat base)

Hemisphere (r)	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3} \pi r^3$	TSA = CSA + one circular base (πr^2)
Frustum of a Cone (R, r, h, l)	$\pi l(R+r)$	$\pi [l(R+r)+R^2+r^2]$	$\frac{1}{3}\pi h(R^2+r^2+Rr)$	$l = \sqrt{(h^2 + (R - r)^2)}$

Surface Area of Combination of Solids

Critical Rule for Surface Area of Combinations: When two solids are joined together, the joined surfaces are NOT included in the total surface area. You only add up the surfaces that are actually visible from outside the combined shape.

Combination	TSA Formula	What's Excluded?
Cone on top of Cylinder	CSA of cone + CSA of cylinder + area of base circle = $\pi r l + 2\pi r h_2 + \pi r^2$ where $l = \sqrt{(r^2 + h_1^2)}$	The circular base where the cone sits on the cylinder (hidden surface)
Hemisphere on Cylinder	CSA of cylinder + CSA of hemisphere + base circle = $2\pi r h + 2\pi r^2 + \pi r^2 = 2\pi r h + 3\pi r^2$	The top circular base of the cylinder (covered by the hemisphere)
Cone on Hemisphere	CSA of cone + CSA of hemisphere = $\pi r l + 2\pi r^2$	The base circle (hemisphere's flat face hidden by the cone)
Cylinder with 2 Hemispherical Ends	CSA of cylinder + CSA of 2 hemispheres = $2\pi r h + 4\pi r^2$	Both circular ends of the cylinder (covered by hemispheres)

Hemisphere on Cube	TSA of cube – base circle of hemisphere + CSA of hemisphere = $6a^2 - \pi r^2 + 2\pi r^2 = 6a^2 + \pi r^2$	The circular region on the cube's face where the hemisphere is attached
Conical Cavity in Cylinder	CSA of cylinder + base circle + CSA of cone = $2\pi r h + \pi r^2 + \pi r l$	The open top surface (the cone is carved inward, creating a cavity)

Volume of Combination of Solids

Golden Rule for Volume of Combinations:

Total Volume = Sum of individual volumes of each component solid.

If a cavity/hollow is present, subtract the volume of the cavity from the outer solid.

Conversion of Solid from One Shape to Another

The Fundamental Principle

When a solid is melted/recast into another shape:

Volume of original solid = Volume of new solid

The surface area and shape change, but the total volume (amount of material) is conserved.

Conversion Type	Key Equation
Sphere melted into cylinders	$4/3\pi R^3 = n\pi r^2 h$ where (n) = number of cylinders
Sphere melted into smaller spheres	$4/3\pi R^3 = 4/3n\pi r^3$
Cylinder melted into cones	$\pi r^2 h = 1/3 n\pi r^2 h$

Cone melted into sphere	$1/3\pi r^2 h = 4/3\pi R^3$
Water from cylinder transferred to cuboid	$\pi r^2 h_{\text{cylinder}} = lbh_{\text{cuboid}}$

Frustum of a Cone: Full Explanation

What is a Frustum?

Take a cone and cut it with a plane parallel to its base. Remove the smaller cone at the top. The remaining solid with two circular faces (one bigger, one smaller) and a curved lateral surface is called the frustum of a cone.

Key Parameters of a Frustum

- R = radius of the larger (bottom) circular base
- r = radius of the smaller (top) circular base
- h = vertical height of the frustum
- l = slant height = $\sqrt{h^2 + (R - r)^2}$

Frustum Formulas

- Slant Height: $l = \sqrt{h^2 + (R - r)^2}$
- CSA: $\pi l(R + r)$
- TSA: $\pi l(R+r) + \pi R^2 + \pi r^2$
- Volume: $\frac{1}{3} \pi h(R^2 + r^2 + Rr)$

Chapter 12 at a Glance

Topic	Key Formula / Principle	Exam Tip
Cone CSA	πrl , where $l = \sqrt{r^2 + h^2}$	Always calculate the slant height (l) first if only (h) is given.

Sphere TSA	$4\pi r^2$ (same as CSA)	TSA and CSA are the same for a sphere because it has no flat base.
Hemisphere TSA	$3\pi r^2 = 2\pi r^2 + \pi r^2$	Exclude the flat circular base when the hemisphere is joined to another solid.
Combination – TSA	Sum of the visible curved and flat surfaces only	Draw the figure and mark hidden surfaces before applying formulas.
Combination – Volume	Sum of the volumes of all component solids	Add all solid volumes and subtract any cavities or hollows.
Conversion of Solids	$\text{Volume}_1 = \text{Volume}_2$	Volume is conserved during melting or reshaping, though surface area may change.
Frustum – Volume	$\frac{1}{3}\pi h(R^2+r^2+Rr)$	Do not forget the middle term (Rr).
Frustum – CSA	$\pi l(R+r)$, where $l = \sqrt{h^2+(R-r)^2}$	If circumferences are given instead of radii, convert them before substituting.
Frustum – TSA	$\pi l(R+r) + \pi R^2 + \pi r^2$	Always include both circular bases while finding TSA.