Chapter 2: Introduction to Linear Polynomials Notes for Class 9

Basics of Algebraic Expressions

Definition:
An algebraic expression is a combination of constants, variables, and arithmetic operations (+, −, ×, ÷, powers).
For example, $2x^2 + 5xy - 3y^2$ is an algebraic expression in variables x and y.

Key Terms:

- Terms: A term is each individual part of an expression separated by addition or subtraction. In $2x^2 + 5xy - 3y^2$, the three terms are $2x^2$, $5xy$, and $-3y^2$.
- Coefficients: A coefficient is the number in front of the variable. In $2x^2$, the coefficient is 2. In $-3y^2$, the coefficient is −3.
- Variables: A variable (like x, y, z, t) is a letter that can take different values. In $3z + 7$, the variable is z.
- Constants: A constant is a fixed number. In $3z + 7$, the constant is 7.

Differences between a Polynomial and a Non-Polynomial

Not every algebraic expression is a polynomial.
A **polynomial in one variable** (also called a univariate polynomial) is an algebraic expression involving only one variable, where all powers of that variable are non-negative integers (0, 1, 2, 3, …). There are no variables in denominators and no variables under square roots.

Expression	Polynomial?	Reason
$x^2 + 5x + 3$	Yes	All powers (2,1,0) are non-negative integers; one variable.

$3y^3 - 4y^2 + 5$	Yes	The highest power is 3, and all exponents are non-negative integers.
$7z + 2$	Yes	Linear polynomial in the variable (z).
8	Yes	Constant polynomial (degree 0).
$(1/x) + 3$	No	The variable is in the denominator, which is equivalent to the negative exponent x^{-1}.
$\sqrt{x} + 2$	No	The variable has a fractional exponent: $\sqrt{x}=x^{1/2}$.
$x^2 + y^2 + 3$	Yes, but not a polynomial in one variable	It is a polynomial in two variables (a **bivariate polynomial**), not a univariate polynomial.

Degree of a Polynomial Explained

Definition:
The degree of a polynomial is the highest power of the variable present in the expression.
For example, in $x^2 + 5x + 3$, the highest power is 2, so the degree is 2.

Classification of Polynomails by Degree

Degree	Name	General Form	Example
0	Constant Polynomial	a $(a \neq 0)$	8, -3
1	Linear Polynomial	$ax+b$ $(a \neq 0)$	3z+7, 2x-5
2	Quadratic Polynomial	ax^2+bx+c $(a \neq 0)$	x^2+5x+3
3	Cubic Polynomial	ax^3+bx^2+cx+d $(a \neq 0)$	$3y^3-4y^2+5$

4	Quartic Polynomial (Fourth-Degree Polynomial)	$ax^4+bx^3+cx^2+dx+e$ ($a \neq 0$)	$x^4-3x^3+6x^2-2x+7$

Concept of Linear Polynomials in One Variable

A linear polynomial is a polynomial of degree 1.

General Form of a Linear Polynomial:
$p(x) = ax + b$
where $a \neq 0$ (a is any non-zero real number), and b is any real number (the constant term). The variable can be x, y, z, t or any letter.

Examples of Linear Polynomials

Linear Polynomial	Variable	Coefficient (a)	Constant (b)
$3z + 7$	z	3	7
$2x - 5$	x	2	(-5)
$50m + 200$	m	50	200
$4x$	x	4	0
$-x + 9$	x	(-1)	9

Why Must $a \neq 0$?
If a were 0, the expression $ax + b$ would reduce to just b a constant with no variable. A constant has degree 0, not degree 1. So the condition $a \neq 0$ is what keeps a linear polynomial ‘linear’ (degree 1).

Evaluating a Linear Polynomial

To evaluate a polynomial means to find its value at a specific number.

Step-by-Step Process

1. Write down the polynomial.
2. Replace every occurrence of the variable with the given value.
3. Apply order of operations: multiplication and division before addition and subtraction.
4. Simplify to get a single number.

Example: Evaluate $p(x) = 5x - 3$
Solution: Find the value of $p(x) = 5x - 3$ at $x = 0$, $x = -1$, and $x = 2$.
$p(0) = 5(0) - 3 = 0 - 3 = -3$
$p(-1) = 5(-1) - 3 = -5 - 3 = -8$
$p(2) = 5(2) - 3 = 10 - 3 = 7$

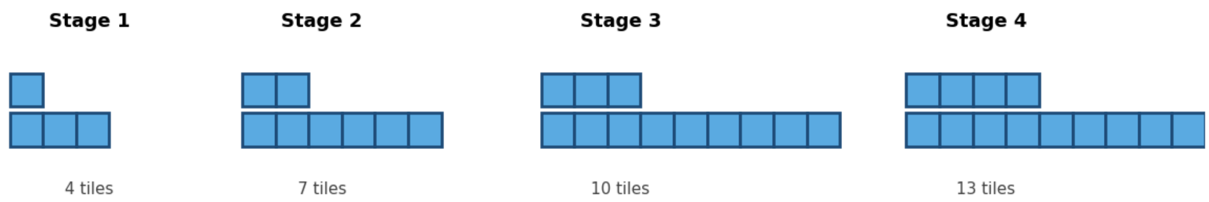
When you set a linear polynomial equal to a specific value, you get a **linear equation**
Example: A bookshop sells notebooks for ₹15 each and adds a ₹30 packaging charge per order. Priya spent ₹165 in total. How many notebooks did she buy?
Solution: Let n = number of notebooks
Total cost = $15n + 30$
Equation: $15n + 30 = 165$
$15n = 165 - 30 = 135$
$n = 135 \div 15 = 9$
Priya bought 9 notebooks.

Identifying Linear Patterns

Definition:

A linear pattern is a sequence in which the difference between any two consecutive terms is always the same. This fixed difference is called the common difference. The general (nth) term of a linear pattern is always a linear polynomial in n.

Example: Triangular tiles are arranged in stages.



(a) How many tiles at Stage 20?
(b) Which stage has 61 tiles?

Solution: Stage 1: 4 tiles Stage 2: 7 tiles Stage 3: 10 tiles Stage 4: 13 tiles
The common difference: $7 - 4 = 3$, $10 - 7 = 3$, $13 - 10 = 3$.
Common difference = 3.
nth term = first term + (n − 1) × common difference $= 4 + (n - 1) \times 3 = 4 + 3n - 3 = 3n + 1$

(a) Number of tiles at Stage 20 $= 3(20) + 1 = 61$ tiles.
(b) Which stage has 61 tiles?
$3n + 1 = 61 \rightarrow 3n = 60 \rightarrow n = 20$

Modeling Linear Growth and Decay with Polynomials

Linear Growth

Occurs when a quantity increases by a fixed amount at each step. The common difference is positive. The graph slopes upward.

Example: A water tank starts with 300 litres and water is pumped in at 150 litres per hour. The amount of water W (in litres) after t hours:
$W(t) = 300 + 150t$

Time (hours), (t)	0	1	2	3	4

Water (litres), (W)	300	450	600	750	900

Each hour the water increases by exactly 150 litres: a constant increase. This is linear growth.

Linear Decay

Occurs when a quantity decreases by a fixed amount at each step. The common difference is negative. The graph slopes downward.

Example: A candle is 30 cm tall when lit. It burns at 2 cm per hour. The height h (in cm) after t hours:

$h(t) = 30 - 2t$

Time (hours), (t)	**0**	**2**	**4**	**6**	**8**
Height (cm), (h)	30	26	22	18	14

Each hour the height decreases by exactly 2 cm: a constant decrease. This is linear decay.

Linear Relationships Between Two Variables

Linear Equation in Two Variables

$ax + by = c$

Here, x and y are two different variables, and a, b, c are constants.

Example: A cyclist travels at a constant speed of 15 km/h. The distance d (in km) covered in t hours is:

$d = 15t$

Here, for every 1 hour increase in t, d increases by exactly 15 km. This is a linear relationship between d and t. If you fix d and solve for t, you get a linear equation.

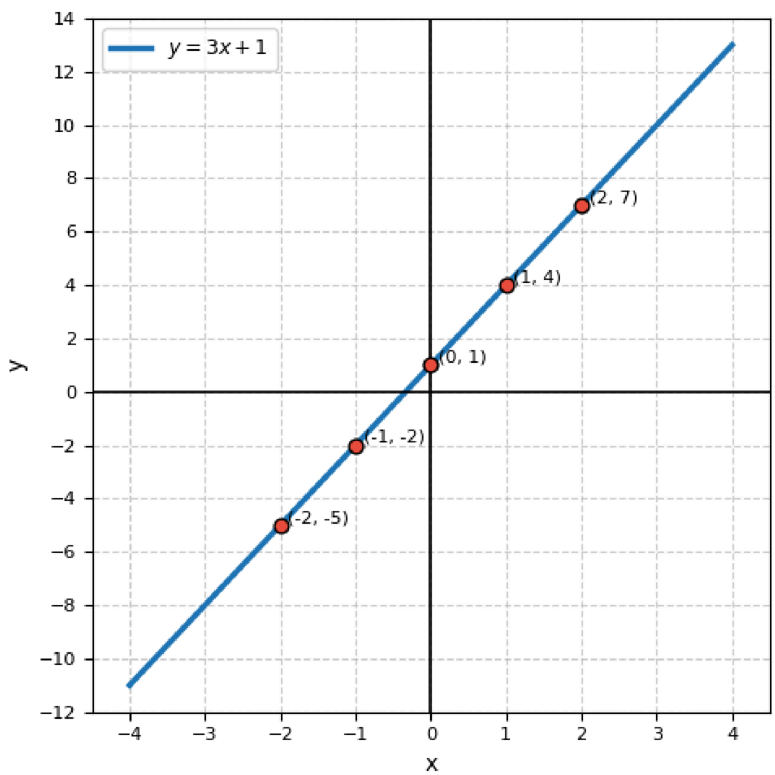
Graphs of Linear Equations

Every linear polynomial, when graphed, gives a perfectly straight line.

How to Plot the Graph of a Linear Polynomial

1. Choose at least 2 values for x. Using x = 0 and x = 1 is usually the easiest start.
2. Substitute each x value into the polynomial to find y.
3. Write these as ordered pairs (x, y).
4. Plot the points on a coordinate plane.
5. Draw a straight line through all three points. If they don't line up, re-check your calculations.

Example 1: Find 3 coordinate pairs for y = 3x + 1



x = 0: y = 3(0) + 1 = 1 → Point: (0, 1)
x = -1: y = 3(-1) + 1 = -2 → Point: (-1, -2)
x = 1: y = 3(1) + 1 = 4 → Point: (1, 4)

The Slope: How Steep Is the Line?

The coefficient a in y = ax + b tells you the slope (steepness) of the line:

- If a > 0 (positive), the line slopes upward from left to right (linear growth)
- If a < 0 (negative), the line slopes downward from left to right (linear decay)
- The larger the value of |a|, the steeper the line
- The constant b is the y-intercept: the point where the line crosses the y-axis (at x = 0)

Example: Compare y = x, y = 2x and y = −x + 1.

Equation	Slope (m)	Y-Intercept (b)	Behavior
y = x	1	0	Increases at a constant rate
y = 2x	2	0	Increases twice as fast as (y = x)
y = -x + 1	-1	1	Decreases as (x) increases