Chapter 4: Quadratic Equations Notes for Class 10

1. What Is a Quadratic Equation

A quadratic equation is any equation of the form:

$ax^2 + bx + c = 0$

where a, b, and c are real numbers, and $a \neq 0$.

Where,

a = coefficient of x^2 (also called the quadratic coefficient). It cannot be zero. If it were, the equation would just be linear.

b = coefficient of x (the linear coefficient)

c = the constant term

Converting to Standard Form

Sometimes a quadratic equation won't look like $ax^2 + bx + c = 0$. You need to rearrange it. For example:

Example: Is $x(x - 4) = -3$ a quadratic equation?

Expand: $x^2 - 4x = -3$

Rearrange: $x^2 - 4x + 3 = 0$

Yes, it's quadratic in standard form with $a = 1$, $b = -4$, $c = 3$.

2. Roots of a Quadratic Equation

The roots (also called solutions or zeroes) of a quadratic equation $ax^2 + bx + c = 0$ are the values of x that make the equation true.

A quadratic equation can have the following:

- Two distinct real roots
- Two equal real roots (also called a repeated root)
- No real roots (the roots are complex/imaginary)

Graphical Meaning

When you plot the quadratic polynomial $y = ax^2 + bx + c$, you get a parabola.

- If the parabola crosses the x-axis at two points ⇒ two distinct real roots
- If it just touches the x-axis at one point ⇒ two equal roots

- If it never touches the x-axis ⇒ no real roots

Roots of a quadratic equation are simply the x-intercepts of the parabola.

3. Solving a Quadratic Equation

3.1 Method 1: Solving by Factorisation

The idea of solving by factorusation is to rewrite the quadratic as a product of two linear factors and then use the zero product property.
Zero Product Property: if $A \times B = 0$, then either $A = 0$ or $B = 0$.

The Split-the-Middle-Term Technique

For $ax^2 + bx + c = 0$:

- Find two numbers p and q such that $p + q = b$ and $p \times q = a \times c$
- Rewrite the middle term using p and q
- Factorise by grouping

Example 1: Solve $2x^2 - 5x + 3 = 0$
Solution: Here, $a = 2$, $b = -5$, $c = 3$. So $a \times c = 6$.
We need two numbers that add to −5 and multiply to 6. That's −2 and −3.
$2x^2 - 2x - 3x + 3 = 0$
$2x(x - 1) - 3(x - 1) = 0$
$\Rightarrow (2x - 3)(x - 1) = 0$
So: $x = 3/2$ or $x = 1$
Example 2: Find two consecutive positive integers whose product is 306.
Solution: Let the integers be x and x + 1.
$x(x + 1) = 306$
$\Rightarrow x^2 + x - 306 = 0$
Find two numbers with sum = 1 and product = −306: 18 and −17
$x^2 + 18x - 17x - 306 = 0$
$x(x + 18) - 17(x + 18) = 0$
$\Rightarrow (x + 18)(x - 17) = 0$
$\Rightarrow x = -18$ (rejected ∵ negative) or $x = 17$
The integers are 17 and 18.

3.2 Method 2: Completing the Square

We transform the equation so that one side becomes a perfect square trinomial of the form $(x + k)^2$.

Steps:

- Write the equation in standard form
- Move the constant to the right side
- If $a \neq 1$, divide throughout by a
- Add (half of the coefficient of $x)^2$ to both sides
- Write the left side as a perfect square
- Take the square root of both sides
- Solve for x

Example 1: Solve $x^2 + 4x - 5 = 0$
Step 1: $x^2 + 4x = 5$
Step 2: Add $(4/2)^2 = 4$ to both sides
$x^2 + 4x + 4 = 5 + 4$
$(x + 2)^2 = 9$
$x + 2 = \pm 3$
$x = 1$ or $x = -5$

3.3 Method 3: The Quadratic Formula

It directly gives you both roots of any quadratic equation.
The Formula:

For $ax^2 + bx + c = 0$ (where $a \neq 0$):

$x = [-b \pm \sqrt{b^2 - 4ac}]/\{2a\}$
That ± means you calculate two values:

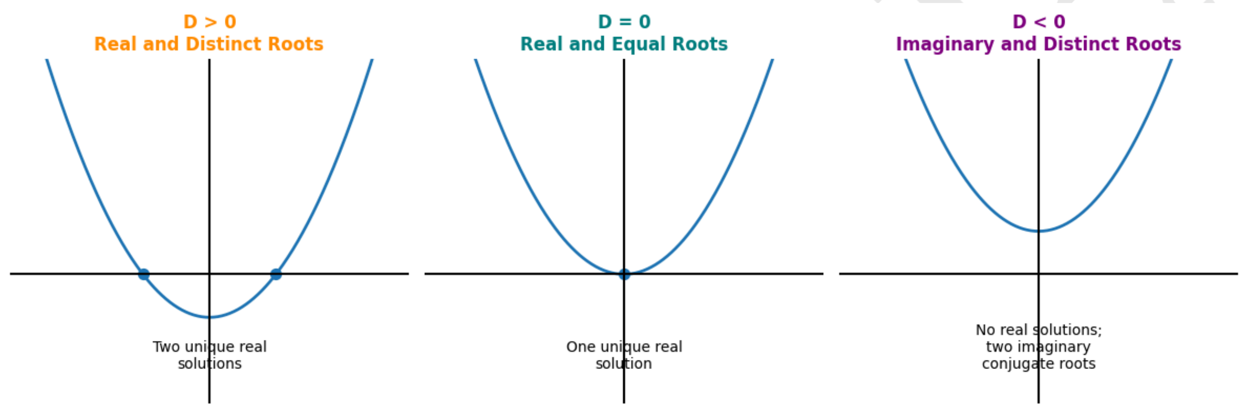
$x_1 = (-b + \sqrt{(b^2 - 4ac)}) / 2a$
$x_2 = (-b - \sqrt{(b^2 - 4ac)}) / 2a$

Example 1: Solve $x^2 - 5x + 6 = 0$
$a = 1, b = -5, c = 6$
Discriminant $= (-5)^2 - 4(1)(6) = 25 - 24 = 1$
$x = (5 \pm \sqrt{1}) / 2 = (5 \pm 1) / 2$
$x = 3$ or $x = 2$

4.The Discriminant

The discriminant, denoted by D (or sometimes Δ).
$D = b^2 - 4ac$
The discriminant tells you exactly what kind of roots to expect.

The Three Cases:



Case 1: $D > 0$ (Positive Discriminant)
The equation has two distinct real roots.
The square root of a positive number is a real number, giving you two different values.
Case 2: $D = 0$ (Zero Discriminant)
The equation has two equal real roots (also called a repeated root).
Both roots are the same: $x = -b/2a$
Case 3: $D < 0$ (Negative Discriminant)
The equation has no real roots.
You'd be taking the square root of a negative number, which has no real solution.

Example 1: Find the nature of roots of $x^2 - 4x + 4 = 0$
$D = (-4)^2 - 4(1)(4) = 16 - 16 = 0$
$\Rightarrow$ Two equal real roots. Both roots $= -(-4)/2(1) = 2$
Example 2: Find the nature of roots of $3x^2 - 5x + 2 = 0$
$D = (-5)^2 - 4(3)(2) = 25 - 24 = 1 > 0$
$\Rightarrow$ Two distinct real roots
Example 3: Find the nature of roots of $x^2 + x + 1 = 0$
$D = (1)^2 - 4(1)(1) = 1 - 4 = -3 < 0$
$\Rightarrow$ No real roots

5. Sum and Product of Roots
If α (alpha) and β (beta) are the two roots of $ax^2 + bx + c = 0$, then:

Sum of roots: $\alpha + \beta = -b/a$
Product of roots: $\alpha \times \beta = c/a$

Example: If one root of $x^2 - 5x + k = 0$ is 2, find k.
Using sum of roots: $2 + \beta = 5$, so $\beta = 3$
Using product of roots: $2 \times 3 = k \Rightarrow k = 6$

6. Important Formulas: Quick Reference Card

Formula	Expression
Standard form	$ax^2 + bx + c = 0$
Quadratic formula	$x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$
Discriminant	$D = b^2 - 4ac$
$D > 0$	Two distinct real roots
$D = 0$	Two equal real roots
$D < 0$	No real roots

Sum of roots ($\alpha + \beta$)	$-b/a$
Product of roots ($\alpha \times \beta$)	c/a
Equal roots condition	$b^2 = 4ac$