Chapter 5: Arithmetic Progression Notes for Class 10

1. What is an Arithmetic Progression?

Definition of Arithmetic Progression
An Arithmetic Progression (AP) is a list of numbers in which each term after the first is obtained by adding a fixed number to the previous term. This fixed number is called the common difference.

General Form of an AP:
a, a+d, a+2d, a+3d, …
where

- a = first term
- d = common difference

How to Find the Common Difference
Common difference = (any term) − (the term before it)

$d = a_n - a_{n-1}$

Example 1: Is 3, 7, 11, 15, 19, … an AP? If yes, find the common difference.
Solution: 7 − 3 = 4, 11 − 7 = 4, 15 − 11 = 4, 19 − 15 = 4
The difference between every pair of consecutive terms is constant = 4.
Yes, it is an AP. Common difference d = 4

Finite and Infinite AP

Type	Description	Example
Finite AP	Has a definite number of terms (has a last term).	2, 5, 8, 11, 14 (5 terms)

Infinite AP	Continues endlessly (has no last term).	2, 5, 8, 11, 14, 17, …

2. nth Term (General Term) of an AP

nth Term Formula

$a_n = a + (n - 1) \times d$

where

- a_n = nth term
- a = first term
- n = term number
- d = common difference

Where Does This Formula Come From?

Let's think about it logically. In an AP starting at a:

1st term = a (add d zero times)
2nd term = a + d (add d once)
3rd term = a + 2d (add d twice)

nth term = a + (n−1)d (add d exactly n−1 times)

Example 1: Find the 10th term of the AP: 2, 7, 12, 17, …

a = 2, d = 7 − 2 = 5, n = 10

$a_{10} = 2 + (10 - 1) \times 5 = 2 + 9 \times 5 = 2 + 45$

$a_{10} = 47$

Example 2: Which term of the AP: 3, 8, 13, 18, … is equal to 78?

a = 3, d = 5. Set $a_n = 78$

$78 = 3 + (n - 1) \times 5$

$\Rightarrow 75 = (n - 1) \times 5 \Rightarrow n - 1 = 15 \Rightarrow n = 16$

78 is the 16th term of the AP.

3. Types of AP & Behaviour of Common Difference

Common Difference	Type	Behaviour	Example
$d > 0$	Increasing AP	Each term is bigger than the last.	2, 6, 10, 14, … ($d = 4$)
$d = 0$	Constant AP	All terms are equal.	5, 5, 5, 5, … ($d = 0$)
$d < 0$	Decreasing AP	Each term is smaller than the last.	20, 15, 10, 5, … ($d = -5$)

4. Sum of n Terms of an AP

The Derivation

$S_n = a + (a+d) + (a+2d) + \ldots + (l-d) + l$ …(1)

$S_n = l + (l-d) + (l-2d) + \ldots + (a+d) + a$ …(2)

Adding (1) and (2): $2S_n = n \times (a + l)$

Since $l = a + (n-1)d$, substituting gives: $S_n = n/2 \times [2a + (n-1)d]$

Sum of First n Terms: when a and d are known

$S_n = n/2 \times [2a + (n - 1)d]$

Use when first term (a) and common difference (d) are given

Sum of First n Terms: when first and last term are known

$S_n = n/2 \times (a + l)$

l = last term = $a + (n-1)d$

Use when you know both ends of the AP

Example:

Find the sum of the first 22 terms of the AP: 8, 3, −2, …

$a = 8,\quad d = 3 - 8 = -5,\quad n = 22$

$\Rightarrow S_{22} = 22/2 \times [2\times 8 + (22-1)\times(-5)] = 11 \times [16 + (21)(-5)]$

$\Rightarrow = 11 \times [16 - 105] = 11 \times (-89)$

$S_{22} = -979$

Important Relationship: $a_n = S_n - S_{n-1}$

The nth term = Sum of n terms − Sum of (n−1) terms

$a_n = S_n - S_{n-1}$ (valid for $n \geq 2$)

Also note: $a_1 = S_1$

Sum of First n Natural Numbers

The natural numbers 1, 2, 3, …, n form an AP with a = 1 and d = 1. So:

Sum of First n Natural Numbers

$S_n = n(n + 1) / 2$

Example: Sum of 1 to 10 = 10 × 11 / 2 = 55

5. Arithmetic Mean

Arithmetic Mean of Two Numbers

A.M. = (a + b) / 2

If a, A, b are in AP, then A is the arithmetic mean of a and b

If three numbers a, b, c are in AP, then b = (a + c) / 2, which means 2b = a + c.

Example 1: If x, 12, y are in AP, and x + y = 24, find x and y.

Solution: Since x, 12, y are in AP: $2 \times 12 = x + y$

$\Rightarrow \quad x + y = 24$

We need another condition. Given x + y = 24, and 12 is the AM of x and y.

⇒ Any pair with sum 24 works, e.g., x = 10, y = 14 with d = 2 or x = 8, y = 16 with d = 4.

6. All Formulas at a Glance

Concept	Formula

First term	a
Common difference	d = a_n - a_{n-1}
General form	a, a+d, a+2d, a+3d,...
nth term (general term)	a_n = a + (n-1)d
Last term ((l))	l = a + (n-1)d
Sum Formula 1 (using d)	S_n = (n/2)[2a + (n-1)d]
Sum Formula 2 (using l)	S_n = (n/2)(a + l)
nth term from (S_n)	a_n = Sn - S{n-1}
Arithmetic Mean of (a) and (b)	AM = (a+b)/2
Sum of first (n) natural numbers	n(n+1)/2
If (a), (b), (c) are in AP	2b = a + c