Chapter 7: Coordinate Geometry Notes for Class 10

1. Introduction to Coordinate Geometry

Coordinate Geometry (also called Cartesian Geometry or Analytical Geometry) is a branch of mathematics that uses a coordinate system, a grid with numbers, to describe the position of every point on a flat surface.

2. The Cartesian Coordinate System

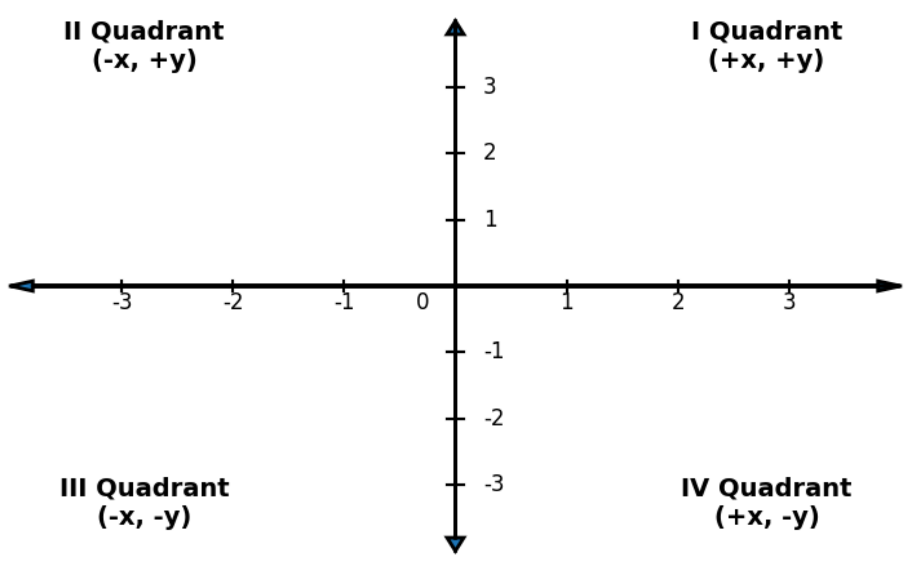
The Cartesian coordinate system is named after the French mathematician René Descartes, who invented it in the 17th century.

The system consists of two perpendicular number lines:

The X-axis: the horizontal line (runs left and right)

The Y-axis: the vertical line (runs up and down)

These two lines intersect at a point called the Origin, denoted by O, with coordinates (0, 0).



Representing a Point

Any point on this plane is written as an ordered pair (x, y), where:

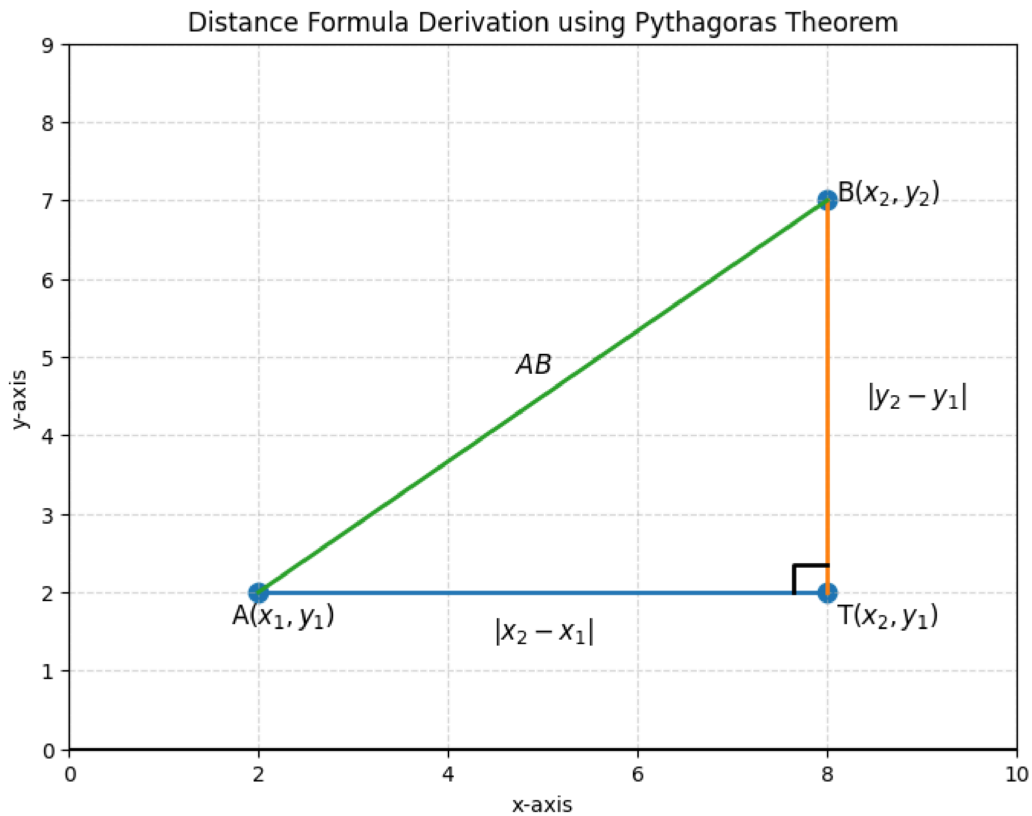
x is the x-coordinate (also called the abscissa): it measures how far the point is from the Y-axis, moving horizontally.
y is the y-coordinate (also called the ordinate): it measures how far the point is from the X-axis, moving vertically.

Example: Point P(3, 2) is 3 units to the right of the Y-axis and 2 units above the X-axis.

3. Quadrants and Sign Convention
The X-axis and Y-axis divide the Cartesian plane into four regions called quadrants.

Quadrant	Location	Sign of x	Sign of y	Example
Quadrant I	Top-right	Positive (+)	Positive (+)	(3, 5)
Quadrant II	Top-left	Negative (−)	Positive (+)	(−2, 4)
Quadrant III	Bottom-left	Negative (−)	Negative (−)	(−3, −1)
Quadrant IV	Bottom-right	Positive (+)	Negative (−)	(6, −2)

4. Distance Formula
The Distance Formula lets you calculate the exact distance between any two points on the Cartesian plane.
Derivation (The Pythagoras Connection)
Let's say you have two points, $A(x_1, y_1)$ and $B(x_2, y_2)$, plotted on the Cartesian plane.
Draw a horizontal line from A and a vertical line from B. They meet at a point $T(x_2, y_1)$. This creates a right-angled triangle ATB, with the right angle at T.



Now apply the Pythagoras Theorem:

$AB^2 = AT^2 + BT^2$

Here:

AT = horizontal distance = $|x_2 - x_1|$
BT = vertical distance = $|y_2 - y_1|$

So:

$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Therefore, the Distance Formula is:
$AB = \sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2]}$.
Solved Example: Find the distance between the points A(4, 3) and B(8, 6).
Solution: Using the distance formula:

$AB = \sqrt{[(8 - 4)^2 + (6 - 3)^2]}$
$AB = \sqrt{[4^2 + 3^2]}$
$AB = \sqrt{[16 + 9]}$
$AB = \sqrt{25}$
AB = 5 units

5. Distance of a Point from the Origin

This is a special case of the distance formula. If one of the points is the Origin O(0, 0) and the other is P(x, y), then:
$OP = \sqrt{[x^2 + y^2]}$.
Example: Distance of the point P(5, 12) from the origin:
$OP = \sqrt{(5^2 + 12^2)} = \sqrt{(25 + 144)} = \sqrt{169} = 13$ units

6. Section Formula (Internal Division)

If point P(x, y) divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio m : n, then:
$P(x, y) = [(mx_2+nx_1)/(m+n), (my_2+ny_1)/(m+n)]$.

Solved Example: Find the coordinates of point P that divides the line segment joining A(2, −2) and B(−7, 4) in the ratio 1:2 internally.
Solution:
Here, $(x_1, y_1) = (2, -2)$, $(x_2, y_2) = (-7, 4)$, m = 1, n = 2
$x = (1 \times (-7) + 2 \times 2) / (1 + 2) = (-7 + 4) / 3 = -3/3 = -1$
$y = (1 \times 4 + 2 \times (-2)) / (1 + 2) = (4 - 4) / 3 = 0/3 = 0$
P = (−1, 0)

7. Midpoint Formula

The midpoint is a special case of the section formula where the ratio is 1 : 1, meaning the point divides the segment into two equal halves.
If M is the midpoint of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$, then:
$M(x, y) = (x_1+x_2)/2, (y_1+y_2)/2$

Simply put: average the x-coordinates, average the y-coordinates.

Solved Example: Find the midpoint of the segment joining P(−3, 3) and Q(1, 4).

Solution:
$x = (-3 + 1) / 2 = -2/2 = -1$
$y = (3 + 4) / 2 = 7/2$
Midpoint = $(-1, 7/2)$

8. Points of Trisection

Trisection means dividing a line segment into three equal parts. If P and Q are the two trisection points of segment AB, then:

P divides AB in the ratio 1 : 2 (one-third from A)
Q divides AB in the ratio 2 : 1 (two-thirds from A, or one-third from B)

Solved Example: Find the points of trisection of the line segment joining A(2, −2) and B(−7, 4).
Solution:
For point P (ratio 1:2):
$x = (1 \times (-7) + 2 \times 2) / (1 + 2) = (-7 + 4) / 3 = -1$
$y = (1 \times 4 + 2 \times (-2)) / 3 = (4 - 4) / 3 = 0$
$P = (-1, 0)$
For point Q (ratio 2:1):
$x = (2 \times (-7) + 1 \times 2) / (2 + 1) = (-14 + 2) / 3 = -4$
$y = (2 \times 4 + 1 \times (-2)) / 3 = (8 - 2) / 3 = 2$
$Q = (-4, 2)$
Points of trisection: (−1, 0) and (−4, 2)

9. Centroid of a Triangle

The centroid is the point where all three medians of a triangle meet. A median connects a vertex to the midpoint of the opposite side. The centroid always divides each median in the ratio 2 : 1 from the vertex.

If a triangle has vertices $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$, its centroid G is:
$G(x, y) = (x_1+x_2+x_3)/3, (y_1+y_2+y_3)/3$

In simple terms: take the average of all three x-coordinates and all three y-coordinates.

Solved Example: Find the centroid of the triangle with vertices A(−1, −3), B(2, 1), and C(8, −4).
Solution:
$x = (-1 + 2 + 8) / 3 = 9/3 = 3$
$y = (-3 + 1 + (-4)) / 3 = -6/3 = -2$
Centroid G = (3, −2)

10. Area of a Triangle Using Coordinates

If the vertices of a triangle are $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$, then:
$\text{Area} = (1/2)|x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)|$

Solved Example: Find the area of the triangle with vertices A(1, 2), B(4, 2), and C(3, 5).
Solution:
$\text{Area} = (1/2)\,|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$
$= (1/2)\,|1(2 - 5) + 4(5 - 2) + 3(2 - 2)|$
$= (1/2)\,|1(-3) + 4(3) + 3(0)|$
$= (1/2)\,|-3 + 12 + 0|$
$= (1/2) \times 9$
Area = 9/2 = 4.5 square units

11. Collinearity of Three Points
Three points are collinear if they all lie on the same straight line.
Method 1: Using the Area Formula:
If Area of △ABC = 0, then A, B, C are collinear.
$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$
Method 2: Using the Distance Formula:
If AB + BC = AC (i.e., the sum of two distances equals the third), then the points are collinear.

Solved Example: Check whether A(3, 0), B(6, 4), and C(−1, 3) are collinear.

Solution (using area method):
Area = (1/2) |3(4 − 3) + 6(3 − 0) + (−1)(0 − 4)|
= (1/2) |3(1) + 6(3) + (−1)(−4)|
= (1/2) |3 + 18 + 4|
= (1/2) × 25
= 12.5 square units ≠ 0
Since area ≠ 0, the points are not collinear.

12. All Formulas at a Glance

Formula	Expression
Distance between $A(x_1,y_1)$ and $B(x_2,y_2)$	$\sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2]}$
Distance from Origin O(0,0) to P(x,y)	$\sqrt{(x^2 + y^2)}$
Section Formula (internal, ratio m:n)	$x = (mx_2 + nx_1)/(m + n)$, $y = (my_2 + ny_1)/(m + n)$
Midpoint Formula	$x = (x_1+x_2)/2$, $y = (y_1+y_2)/2$
Centroid of Triangle	$G = ((x_1+x_2+x_3)/3, (y_1+y_2+y_3)/3)$
Area of Triangle	(1/2)
Collinearity Condition	Area = 0