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# Class 10 Maths Chapter 3: Pair of Linear Equations in Two Variables

**1. What is a Linear Equation in Two Variables?**

Before we get to pairs of equations, let us revisit the building block.

A linear equation in two variables is any equation of the form:

$ax + by + c = 0$

where:

- x and y are the two variables (the unknowns we want to find)
- a, b, and c are real numbers (constants)
- a and b are not both zero simultaneously (at least one must be non-zero)

When you plot this equation on a graph, you always get a straight line.

Examples of linear equations in two variables:

$2x + 3y = 12$

$x - y + 5 = 0$

$4x = 7y - 3$

**2. What is a Pair of Linear Equations in Two Variables?**

When we have two linear equations in the same two variables (x and y), and we want to find values of x and y that satisfy both equations at the same time, we call it a system or pair of linear equations in two variables.

**3. General Form and Key Terminology**

**General Form**

The standard (general) form of a pair of linear equations in two variables is:

$a_1x + b_1y + c_1 = 0 \ldots (1)$

$a_2x + b_2y + c_2 = 0 \ldots (2)$

where $a_1, a_2, b_1, b_2, c_1, c_2$ are all real numbers, and:

- $a_1^2 + b_1^2 \neq 0$ (meaning $a_1$ and $b_1$ are not both zero)
- $a_2^2 + b_2^2 \neq 0$ (meaning $a_2$ and $b_2$ are not both zero)

**Key Terms**

| Term | Meaning |
|---|---|
| **Variables** | The unknowns, usually x and y |
| **Coefficients** | The numbers multiplied with variables ($a_1$, $b_1$, $a_2$, $b_2$) |
| **Constants** | The fixed numbers ($c_1$, $c_2$) |
| **Solution** | A pair (x, y) that satisfies both equations simultaneously |
| **Consistent system** | A system that has at least one solution |
| **Inconsistent system** | A system that has no solution |
| **Dependent system** | A consistent system with infinitely many solutions |

**4. Graphical Method of Solution**

Every linear equation in two variables represents a straight line on the coordinate plane. So a pair of two linear equations represents two straight lines.

**Steps to Solve Graphically:**

- Rewrite each equation in the slope-intercept form $y = mx + c$ .
- For each equation, find at least two coordinate points by substituting values of x.
- Plot both lines on graph paper.
- The point where the two lines intersect is the solution.

Worked Example

Solve graphically: $2x + y - 6 = 0$ and $4x - 2y - 4 = 0$

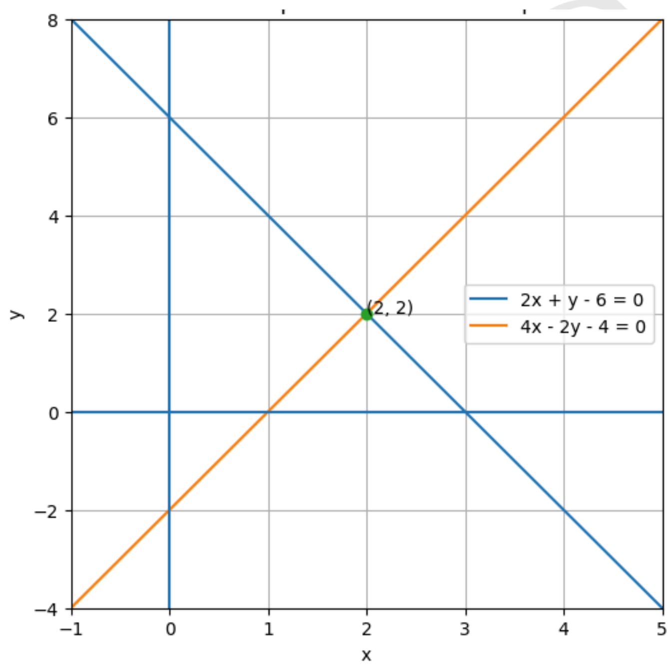
For 2x + y − 6 = 0 ⇒ y = 6 − 2x:

| x | y |
|---|---|
| 0 | 6 |
| 3 | 0 |

For 4x − 2y − 4 = 0 ⇒ y = 2x − 2:

| x | y |
|---|---|
| 0 | -2 |
| 1 | 0 |

Plotting these and extending the lines, they intersect at the point (2, 2).




So x = 2 and y = 2 is the solution.

**5. Types of Lines and Consistency Conditions**

When two straight lines are drawn on a plane, exactly one of three things can happen:

**Case 1: Lines Intersect at Exactly One Point**
The two lines cross at a single point. This gives exactly one solution: the coordinates of the point of intersection. The system is consistent.
**Condition: $a_1/a_2 \neq b_1/b_2$**
Example: $x + y = 5$ and $x - y = 1$ → intersect at (3, 2)

**Case 2: Lines are Parallel (No Intersection)**
The two lines never meet. There is no solution. The system is inconsistent.
**Condition: $a_1/a_2 = b_1/b_2 \neq c_1/c_2$**
Example: $2x + y = 4$ and $4x + 2y = 9$
Check: $a_1/a_2 = 2/4 = 1/2$, $b_1/b_2 = 1/2 = 1/2$, $c_1/c_2 = 4/9 \neq 1/2$
Since $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ → parallel lines → no solution.

**Case 3: Lines are Coincident (Same Line)**
When the two equations actually represent the same straight line, every point on the line is a solution, giving infinitely many solutions. The system is consistent and dependent.
**Condition: $a_1/a_2 = b_1/b_2 = c_1/c_2$**
Example: $2x + 4y = 8$ and $x + 2y = 4$
Check: $2/1 = 4/2 = 8/4$ ⇒ all ratios equal ⇒ coincident lines ⇒ infinite solutions.

| Condition | Type of Lines | Number of Solutions | System |
|---|---|---|---|
| $a_1/a_2 \neq b_1/b_2$ | Intersecting | Unique (exactly one) | Consistent |
| $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ | Parallel | No solution | Inconsistent |

| | | | |
|---|---|---|---|
| $a_1/a_2 = b_1/b_2 = c_1/c_2$ | Coincident | Infinitely many | Consistent (Dependent) |

**6. Algebraic Methods of Solution**

Algebraic methods give exact solutions and are preferred over the graphical method for board exam problems.

**6.1 Substitution Method**

Steps:

- From either equation, express one variable in terms of the other.
- Substitute this expression into the other equation.
- Solve the resulting single-variable equation.
- Substitute back to find the value of the other variable.
- Write the solution as an ordered pair (x, y).

Solved Example

Solve: $y - 2x = 1$ and $x + 2y = 12$

Step 1: From equation (1): $y = 2x + 1$

Step 2: Substitute into equation (2):

$x + 2(2x + 1) = 12$

$\Rightarrow x + 4x + 2 = 12$

$\Rightarrow 5x = 10$

$\Rightarrow x = 2$

Step 3: Substitute $x = 2$ back into $y = 2x + 1$:

$y = 2(2) + 1 = 5$

Solution: (2, 5)

Verify: $y - 2x = 5 - 4 = 1$ and $x + 2y = 2 + 10 = 12$.

**6.2 Elimination Method**

Here, we eliminate one variable by making its coefficients equal in both equations, then adding or subtracting.

Steps:

- Multiply one or both equations by suitable constants so that the coefficients of one variable become equal.

- Add or subtract the equations to eliminate that variable.
- Solve the resulting equation for the remaining variable.
- Substitute back to find the other variable.

Solved Example
Solve: $x + 2y = 8$ and $2x - 3y = 2$
Step 1: Multiply equation (1) by 2:
$2x + 4y = 16 \ldots (3)$
Step 2: Subtract equation (2) from equation (3):

$2x + 4y = 16$
$2x - 3y = 2$
(−) (+) (−)

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$7y = 14$
$\Rightarrow y = 2$
Step 3: Substitute $y = 2$ into equation (1):
$x + 4 = 8 \Rightarrow x = 4$
Solution: (4, 2)

**6.3 Cross-Multiplication Method**
The Formula
For the system:
$a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$
The solution is given by:
$$x / (b_1c_2 - b_2c_1) = y / (c_1a_2 - c_2a_1) = 1 / (a_1b_2 - a_2b_1)$$
This gives:

$$x = (b_1c_2 - b_2c_1) / (a_1b_2 - a_2b_1)$$
$$y = (c_1a_2 - c_2a_1) / (a_1b_2 - a_2b_1)$$

Condition: $a_1b_2 - a_2b_1 \neq 0$ (otherwise the lines are parallel or coincident)

**Memory Aid**

Write the coefficients in two rows. Cross-multiply diagonally:

**$b_1$ $c_1$ $a_1$ $b_1$**
**$b_2$ $c_2$ $a_2$ $b_2$**

- For x: $(b_1 \times c_2) - (b_2 \times c_1)$
- For y: $(c_1 \times a_2) - (c_2 \times a_1)$
- Denominator: $(a_1 \times b_2) - (a_2 \times b_1)$

Solved Example

Solve: $2x + 3y + 5 = 0$ and $4x - y - 3 = 0$

Here: $a_1 = 2$, $b_1 = 3$, $c_1 = 5$, $a_2 = 4$, $b_2 = -1$, $c_2 = -3$

- Numerator for x: $b_1c_2 - b_2c_1 = (3)(-3) - (-1)(5) = -9 + 5 = -4$
- Numerator for y: $c_1a_2 - c_2a_1 = (5)(4) - (-3)(2) = 20 + 6 = 26$
- Denominator: $a_1b_2 - a_2b_1 = (2)(-1) - (4)(3) = -2 - 12 = -14$

So: $x = -4/-14 = 2/7$ and $y = 26/-14 = -13/7$

Solution: $(2/7, -13/7)$

**7. Equations Reducible to a Pair of Linear Equations**
Sometimes equations may look non-linear, but they can be transformed into a standard linear pair using substitution.

Equations involving $1/x$ and $1/y$ are the most common type in this section.
Example: Solve $2/x + 3/y = 4$ and $5/x - 4/y = 9$
Step 1: Substitute $1/x = u$ and $1/y = v$
The system becomes:
$2u + 3v = 4$ … (1)
$5u - 4v = 9$ … (2)

Step 2: This is now a standard linear pair! Solve using elimination.
Multiply (1) by 4 and (2) by 3:
$8u + 12v = 16$ … (3)
$15u - 12v = 27$ … (4)

Add (3) and (4):
$23u = 43 \Rightarrow u = 43/23$
Substitute back: $2(43/23) + 3v = 4 \Rightarrow v = (4 - 86/23)/3 = 2/23$

Step 3: Back-substitute:
$u = 1/x = 43/23 \Rightarrow x = 23/43$
$v = 1/y = 2/23 \Rightarrow y = 23/2$

**8. Important Formulas at a Glance**

| Concept | Formula/Condition |
|---|---|
| General form | $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ |
| Unique solution | $a_1/a_2 \neq b_1/b_2$ |
| No solution | $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ |
| Infinite solutions | $a_1/a_2 = b_1/b_2 = c_1/c_2$ |
| Cross-multiplication (x) | $x = (b_1c_2 - b_2c_1) / (a_1b_2 - a_2b_1)$ |
| Cross-multiplication (y) | $y = (c_1a_2 - c_2a_1) / (a_1b_2 - a_2b_1)$ |
| Verification | Substitute (x, y) back into both original equations |