Class 10 Maths Chapter 8 Introduction to Trigonometry Notes

Class 10 Maths Chapter 8 Introduction to Trigonometry Notes Free PDF Download is prepared based on the latest CBSE and NCERT syllabus. These notes will help in school exams, board exams and quick revision. They help students to understand the chapter clearly, revise faster and prepare for exams with confidence.

Trigonometry deals with the relationship between the sides and angles of a right-angled triangle. In Class 10, you study trigonometry specifically for acute angles angles between 0° and 90°.

Chapter 8 covers four main topics:

- Trigonometric ratios of acute angles
- Trigonometric ratios of specific angles (0°, 30°, 45°, 60°, 90°)
- 00Trigonometric ratios of complementary angles
- Trigonometric identities

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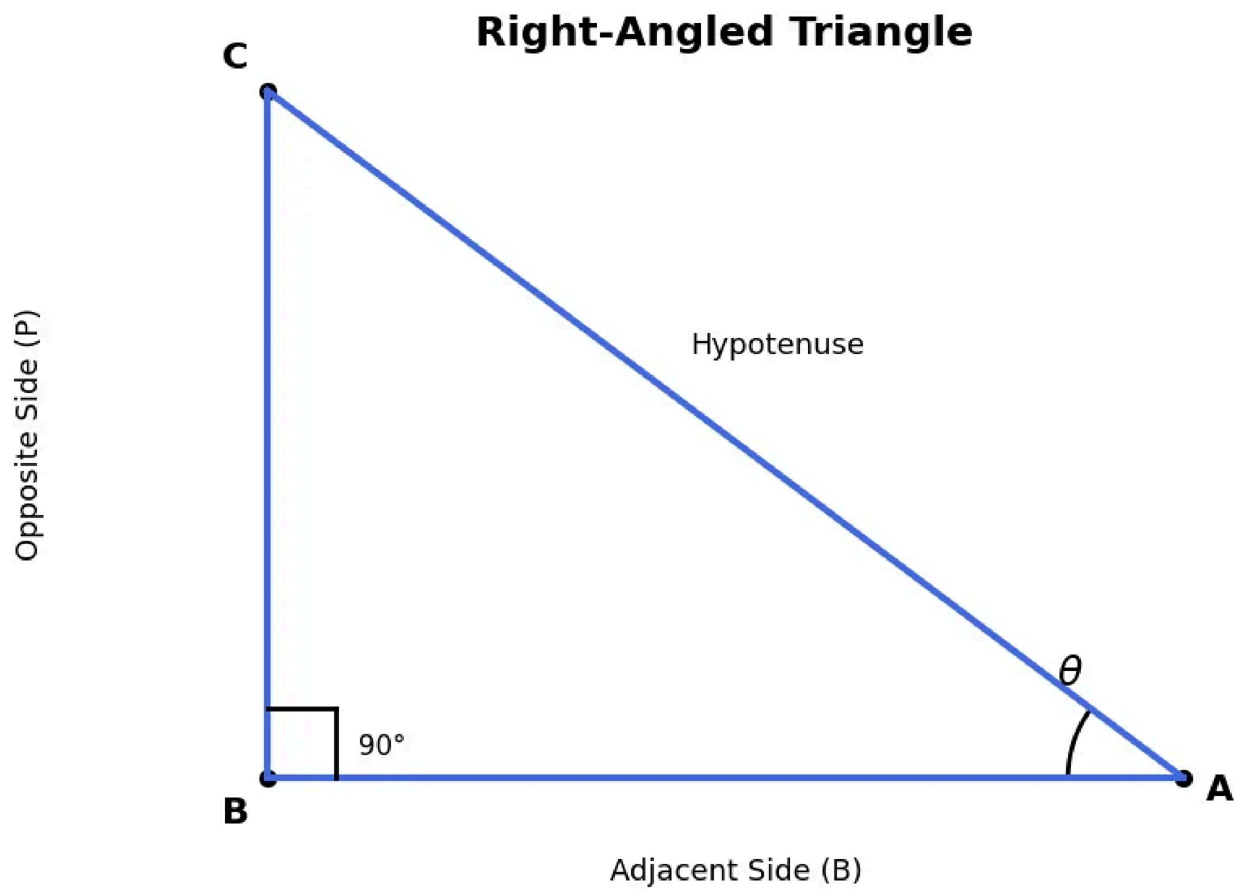
Right Angled Triangle Parts and Terms

A right-angled triangle has one angle equal to 90° (called the right angle).



∠B = 90°, ∠A = θ (reference angle)

∠C = remaining angle

Three sides explained:

Hypotenuse (H): The side opposite to the right angle. Always the longest side. In the diagram, AC is the hypotenuse.

Opposite side (P): The side directly opposite to the reference angle θ. In the diagram, BC is opposite to angle A.

Adjacent side (B): The side next to the reference angle (other than hypotenuse). In the diagram, AB is adjacent to angle A.

Important: The names of sides change when your reference angle changes. The hypotenuse always stays the same, but opposite and adjacent shift based on which angle you're working with.

Trigonometric Ratios of Acute Angles

For a right-angled triangle with reference angle θ, the six trigonometric ratios are:

The six ratios are:

1. Sine (sin θ): sin θ = Opposite/Hypotenuse = P/H = BC/AC
2. Cosine (cos θ): cos θ = Adjacent/Hypotenuse = B/H = AB/AC
3. Tangent (tan θ): tan θ = Opposite/Adjacent = P/B = BC/AB
4. Cosecant (cosec θ): cosec θ = Hypotenuse/Opposite = H/P = AC/BC

(Reciprocal of sin θ)

5. Secant (sec θ): sec θ = Hypotenuse/Adjacent = H/B = AC/AB

(Reciprocal of cos θ)

6. Cotangent (cot θ): cot θ = Adjacent/Opposite = B/P = AB/BC

(Reciprocal of tan θ)

SOH CAH TOA:

SOH → Sin = Opposite/Hypotenuse

CAH → Cos = Adjacent/Hypotenuse

TOA → Tan = Opposite/Adjacent

Reciprocal relationships:

sin θ × cosec θ = 1

cos θ × sec θ = 1

tan θ × cot θ = 1

Quotient relationships:

tan θ = sin θ / cos θ

cot θ = cos θ / sin θ

Formula Table:

Ratio	**Formula**	**Reciprocal**

sin θ	P/H	cosec θ
cos θ	B/H	sec θ
tan θ	P/B	cot θ

Trigonometric Ratios of Specific Angles

These are the standard angle values you must memorize for your board exam.

Standard Values Table:

Angle	**0°**	**30°**	**45°**	**60°**	**90°**
sin	0	1/√2	1/√2	√3/2	1
cos	1	√3/2	1/√2	1/2	0
tan	0	1/√3	1	√3	ND
cosec	ND	2	√2	2/√3	1
sec	1	2/√3	√2	2	ND
cot	ND	√3	1	1/√3	0

ND = Not Defined

sin values:

sin 0° = √0/2 = 0

sin 30° = √1/2 = 1/2

sin 45° = √2/2 = 1/√2

sin 60° = √3/2

sin 90° = √4/2 = 1

Pattern: Numerators go √0, √1, √2, √3, √4

Denominator is always 2

For cos simply reverse the sin pattern:

cos 0° = 1 = sin 90°

cos 30° = √3/2 = sin 60°

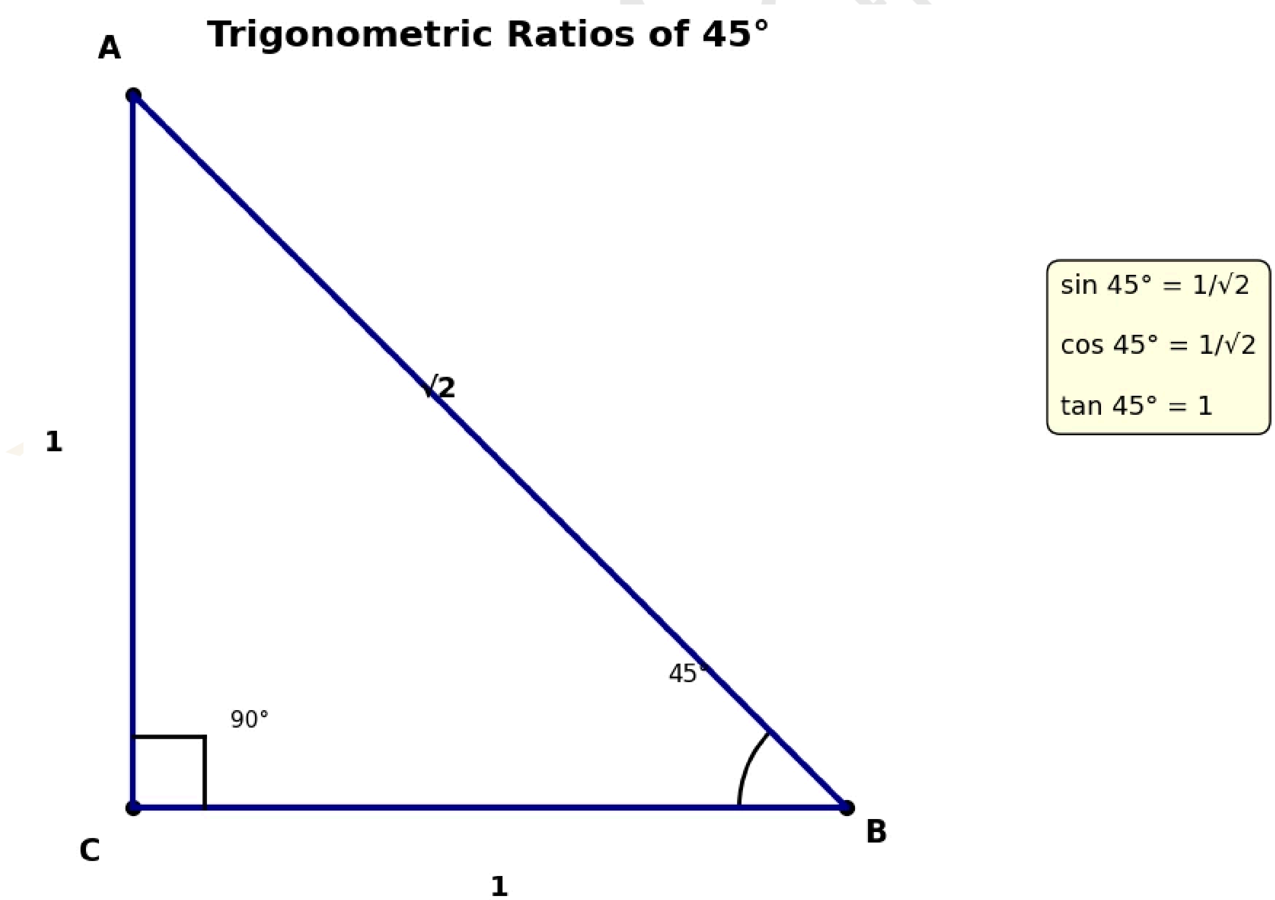
cos 45° = 1/√2 = sin 45°

cos 60° = 1/2 = sin 30°

cos 90° = 0 = sin 0°

How 45° values are derived:

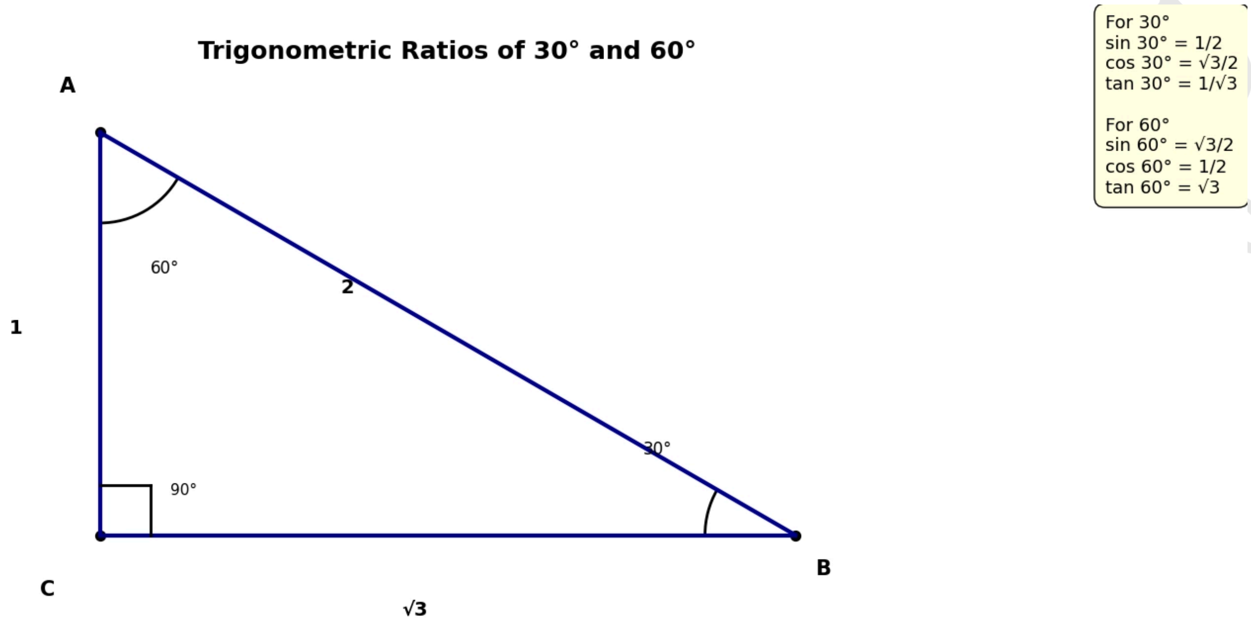
Using an isosceles right triangle with equal sides = 1:



sin 45° = 1/√2, cos 45° = 1/√2, tan 45° = 1

How 30° and 60° values are derived:

Using half of an equilateral triangle with side = 2:



For 30°: sin 30° = 1/2, cos 30° = √3/2, tan 30° = 1/√3

For 60°: sin 60° = √3/2, cos 60° = 1/2, tan 60° = √3

Trigonometric Ratios of Complementary Angles

Two angles are complementary when they add up to 90°.

In a right-angled triangle, the two acute angles are always complementary. This creates a beautiful relationship between trigonometric ratios.

The Six Complementary Relationships:

sin(90° - θ) = cos θ

cos(90° - θ) = sin θ

tan(90° - θ) = cot θ

cot(90° - θ) = tan θ

sec(90° - θ) = cosec θ

cosec(90° - θ) = sec θ

The prefix "co" literally means complement.

sin ↔ cosine (co-sine)

tan ↔ cotangent (co-tangent)

sec ↔ cosecant (co-secant)

Proof of sin(90° - θ) = cos θ:

In right △ABC with ∠B = 90°, ∠A = θ:

Then ∠C = 90° - θ

For angle A (= θ):

sin θ = BC/AC

For angle C (= 90° - θ):

sin(90° - θ) = AB/AC (opposite to C is AB)

= B/H

= cos θ

Practical example:

sin 67° = sin(90° - 23°) = cos 23°

tan 40° = tan(90° - 50°) = cot 50°

sec 15° = sec(90° - 75°) = cosec 75°

Trigonometric Identities for Class 10

A trigonometric identity is an equation involving trigonometric ratios that holds true for all values of the angle.

Identity 1:

$\sin^2\theta + \cos^2\theta = 1$

Proof using Pythagoras:

In right △ABC: $AC^2 = AB^2 + BC^2$

Dividing both sides by AC^2:

$AB^2/AC^2 + BC^2/AC^2 = 1$

$\cos^2\theta + \sin^2\theta = 1$

Derived forms:

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

Identity 2:

$1 + \tan^2\theta = \sec^2\theta$

Proof:

Dividing $AC^2 = AB^2 + BC^2$ by AB^2:

$(AC/AB)^2 = 1 + (BC/AB)^2$

$\sec^2\theta = 1 + \tan^2\theta$

Derived forms:

$\sec^2\theta - \tan^2\theta = 1$

$\tan^2\theta = \sec^2\theta - 1$

Identity 3:

$1 + \cot^2\theta = \text{cosec}^2\theta$

Proof:

Dividing $AC^2 = AB^2 + BC^2$ by BC^2:

$(AC/BC)^2 = (AB/BC)^2 + 1$

$\text{cosec}^2\theta = \cot^2\theta + 1$

Derived forms:

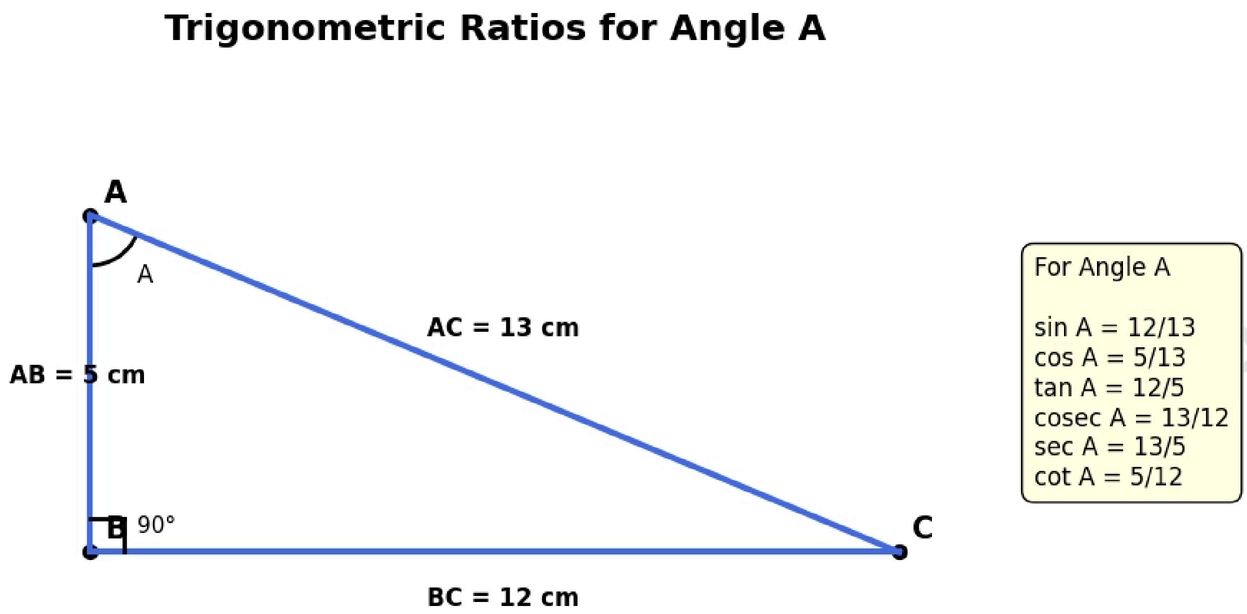
$\text{cosec}^2\theta - \cot^2\theta = 1$

$\cot^2\theta = \text{cosec}^2\theta - 1$

Solved Examples On Chapter 8 Introduction to Trigonometry

Example 1: Finding All Six Ratios

Question: In $\triangle ABC$, right-angled at B, AB = 5 cm and BC = 12 cm. Find all six trigonometric ratios for angle A.



Solution:

Step 1: Find hypotenuse

$AC^2 = AB^2 + BC^2 = 25 + 144 = 169$

AC = 13 cm

Step 2: Identify sides for angle A

Opposite = BC = 12

Adjacent = AB = 5

Hypotenuse = AC = 13

Step 3: Write ratios

sin A = 12/13 cosec A = 13/12

cos A = 5/13 sec A = 13/5

tan A = 12/5 cot A = 5/12

Example 2: Standard Angle Calculation

Question: Evaluate: $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

Solution:

tan 45° = 1, cos 30° = √3/2, sin 60° = √3/2

$= 2(1)^2 + (\sqrt{3}/2)^2 - (\sqrt{3}/2)^2$

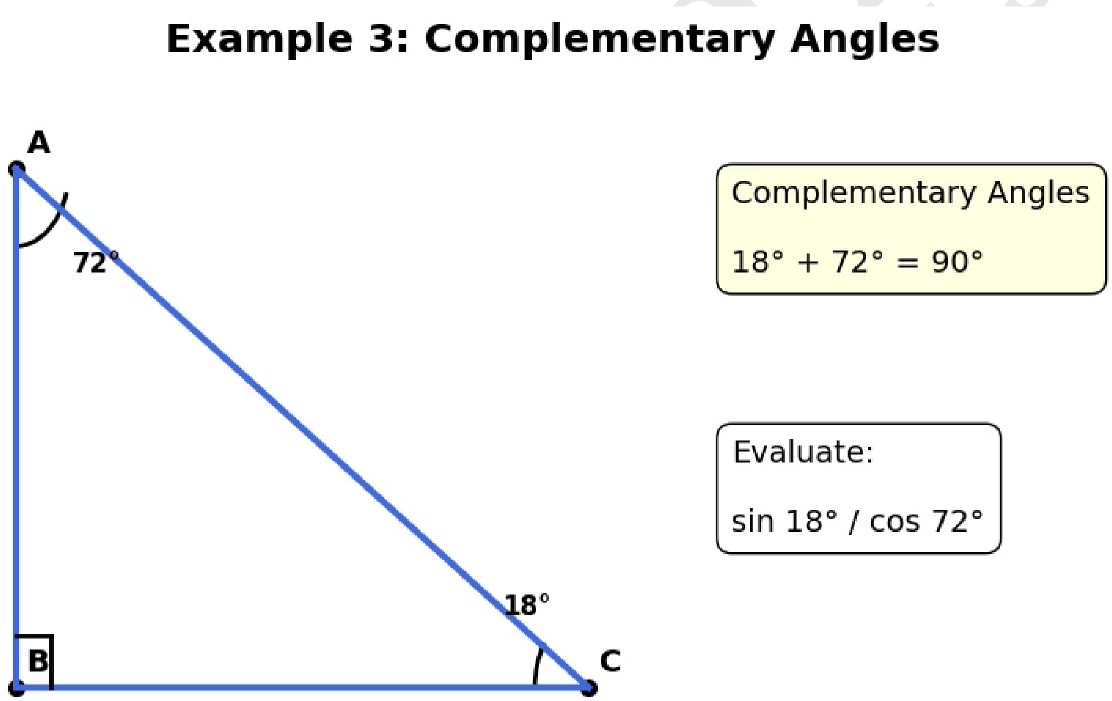
= 2 + 3/4 - 3/4

= 2

Answer: 2

Example 3: Complementary Angles

Question: Evaluate: sin 18°/cos 72°



Solution:

cos 72° = cos(90° - 18°) = sin 18°

sin 18°/cos 72° = sin 18°/sin 18° = 1

Answer: 1

Example 4: Identity Proof

Question: Prove that $(\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

Solution:

LHS = $\sin^2\theta + 2\sin\theta \operatorname{cosec}\theta + \operatorname{cosec}^2\theta$

$+ \cos^2\theta + 2\cos\theta \sec\theta + \sec^2\theta$

Since $\sin\theta \times \operatorname{cosec}\theta = 1$ and $\cos\theta \times \sec\theta = 1$:

$= (\sin^2\theta + \cos^2\theta) + 2 + 2 + \operatorname{cosec}^2\theta + \sec^2\theta$

$= 1 + 4 + (1 + \cot^2\theta) + (1 + \tan^2\theta)$

$= 1 + 4 + 1 + \cot^2\theta + 1 + \tan^2\theta$

$= 7 + \tan^2\theta + \cot^2\theta$ = RHS

Key Formulas

Six Ratios:

$\sin \theta = P/H$ $\cos \theta = B/H$ $\tan \theta = P/B$

$\operatorname{cosec} \theta = H/P$ $\sec \theta = H/B$ $\cot \theta = B/P$

Three Identities:

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

Complementary Pairs:

$\sin(90° - \theta) = \cos \theta$

$\tan(90° - \theta) = \cot \theta$

$\sec(90° - \theta) = \operatorname{cosec} \theta$

Practice Questions on Chapter 8 Introduction to Trigonometry

Section 1: Trigonometric Ratios

Q1: In △PQR, right-angled at Q, PQ = 3 cm, QR = 4 cm. Find sin P, cos P, tan P.

Q2: If cos A = 7/25, find sin A and tan A.

Q3: If sin θ = a/b, find cos θ in terms of a and b.

Q4: Find the value of sin 45° + cos 45°.

Q5: If tan θ = 5/12, find sec θ.

Section 2: Standard Angles

Q6: Evaluate: $\sin^2 30° + \cos^2 30°$

Q7: Find the value of: tan 45° × sec 60° - cosec 30°

Q8: If A = 30°, verify that sin 2A = 2 sin A cos A.

Q9: Evaluate: (cos 0° + sin 30° + sin 45°)(sin 90° - cos 45° + cos 60°)

Q10: Find: $4/3 \tan^2 30° + \sin^2 60° - 3\cos^2 60° + 3/4 \tan^2 60°$

Section 3: Complementary Angles

Q11: Express sin 67° + cos 75° in terms of angles between 0° and 45°.

Q12: Prove: sin 35° cos 55° + cos 35° sin 55° = 1

Q13: Evaluate: (tan 26°)/(cot 64°)

Q14: If sin 3A = cos(A - 26°) where 3A is acute, find A.

Q15: Show that tan 48° tan 23° tan 42° tan 67° = 1.

Section 4: Identities

Q16: Prove: $(1 + \tan^2\theta)/(1 + \cot^2\theta) = \tan^2\theta$

Q17: Prove: (cos A - sin A + 1)/(cos A + sin A - 1) = cosec A + cot A

Q18: If cosec θ - cot θ = 1/3, find cosec θ + cot θ.

Q19: Prove: √(1 + sin θ)/(1 - sin θ) = sec θ + tan θ

Q20: If tan θ + 1/tan θ = 2, find the value of $\tan^2\theta + 1/\tan^2\theta$.

Answers to Practice Questions

Section 1:

A1:

Hypotenuse PR = $\sqrt{(3^2 + 4^2)} = 5$
sin P = QR/PR = 4/5
cos P = PQ/PR = 3/5
tan P = QR/PQ = 4/3

A2:

cos A = 7/25 → B = 7, H = 25
P = $\sqrt{(625 - 49)} = \sqrt{576} = 24$
sin A = 24/25
tan A = 24/7

A3:

$\cos \theta = \sqrt{(1 - \sin^2\theta)} = \sqrt{(1 - a^2/b^2)} = \sqrt{(b^2 - a^2)}/b$

A4:

$\sin 45° + \cos 45° = 1/\sqrt{2} + 1/\sqrt{2} = 2/\sqrt{2} = \sqrt{2}$

A5:

tan θ = 5/12 → P = 5, B = 12, H = 13
sec θ = H/B = 13/12

Section 2:

A6:

$\sin^2 30° + \cos^2 30° = (1/2)^2 + (\sqrt{3}/2)^2 = 1/4 + 3/4 = 1$

A7:

tan 45° × sec 60° - cosec 30°
= 1 × 2 - 2
= 0

A8:

LHS = $\sin 60° = \sqrt{3}/2$
RHS = $2 \sin 30° \cos 30° = 2 \times (1/2) \times (\sqrt{3}/2) = \sqrt{3}/2$
LHS = RHS

A10:

$= 4/3 \times (1/\sqrt{3})^2 + (\sqrt{3}/2)^2 - 3(1/2)^2 + 3/4 \times (\sqrt{3})^2$
$= 4/3 \times 1/3 + 3/4 - 3/4 + 3/4 \times 3$
$= 4/9 + 0 + 9/4$
$= 16/36 + 81/36$
$= 97/36$

Section 3:

A11:

$\sin 67° = \sin(90° - 23°) = \cos 23°$
$\cos 75° = \cos(90° - 15°) = \sin 15°$
Answer = $\cos 23° + \sin 15°$

A13:

$\tan 26° = \tan(90° - 64°) = \cot 64°$
So: $\tan 26°/\cot 64° = \cot 64°/\cot 64° = 1$

A14:

$\sin 3A = \cos(A - 26°)$
$\cos(90° - 3A) = \cos(A - 26°)$
$90° - 3A = A - 26°$
$116° = 4A$
$A = 29°$

A15:

tan 48° = cot(90° - 48°) = cot 42°
tan 23° = cot(90° - 23°) = cot 67°
So: tan 48° × tan 23° × tan 42° × tan 67°
= cot 42° × cot 67° × tan 42° × tan 67°
= (tan 42° × cot 42°) × (tan 67° × cot 67°)
= 1 × 1 = 1

Section 4:

A16:

LHS = $(1 + \tan^2\theta)/(1 + \cot^2\theta)$
= $\sec^2\theta/\csc^2\theta$
= $(1/\cos^2\theta)/(1/\sin^2\theta)$
= $\sin^2\theta/\cos^2\theta$
= $\tan^2\theta$ = RHS

A18: cosec θ - cot θ = 1/3

We know: (cosec θ - cot θ)(cosec θ + cot θ) = $\text{cosec}^2\theta - \cot^2\theta = 1$
So: (1/3)(cosec θ + cot θ) = 1
cosec θ + cot θ = 3

A20: $(\tan\theta + 1/\tan\theta)^2 = 4$

$\tan^2\theta + 2 + 1/\tan^2\theta = 4$
$\tan^2\theta + 1/\tan^2\theta = 2$