Class 9 Maths Chapter 3 The World of Numbers Notes

Class 9 Maths Chapter 3 The World of Numbers Notes Free PDF Download is prepared based on the latest CBSE and NCERT syllabus. These notes will help in school exams, board exams, and quick revision. They help students understand the chapter clearly, revise faster, and prepare for exams with confidence.

Introduction to The World of Numbers

What Is the Number System?

The number system is an organised framework that classifies all numbers into groups based on their properties. Just like objects in the real world are sorted into categories, numbers are sorted into sets natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers. Each set contains the previous one, forming a nested structure.

Importance of Numbers

Numbers are the building blocks of all mathematics. Without understanding how different numbers are related, it becomes impossible to work in algebra, geometry, or any higher topic. Chapter 3 of Class 9 Maths lays this foundation clearly so that students can move confidently into the rest of the course.

Key Terms to Remember

Natural Numbers

Natural numbers are the counting numbers: 1, 2, 3, 4, 5… They start at 1 and go on forever. They do not include zero, negative numbers, or fractions. Symbol: $\mathbb{N}$.

Whole Numbers

Whole numbers are natural numbers with the addition of zero: 0, 1, 2, 3, 4… The only difference from natural numbers is the presence of 0. Symbol: W.

Integers

Integers include all whole numbers and all negative whole numbers: …−3, −2, −1, 0, 1, 2, 3… They do not include fractions or decimals. Symbol: $\mathbb{Z}$.

Rational Numbers

A rational number is any number that can be written as a fraction p/q, where p and q are both integers and q is not zero. The decimal form of a rational number either terminates (ends) or repeats in a pattern. Symbol: ℚ. Examples: ½, −4, 0.75, 0.333…

Irrational Numbers

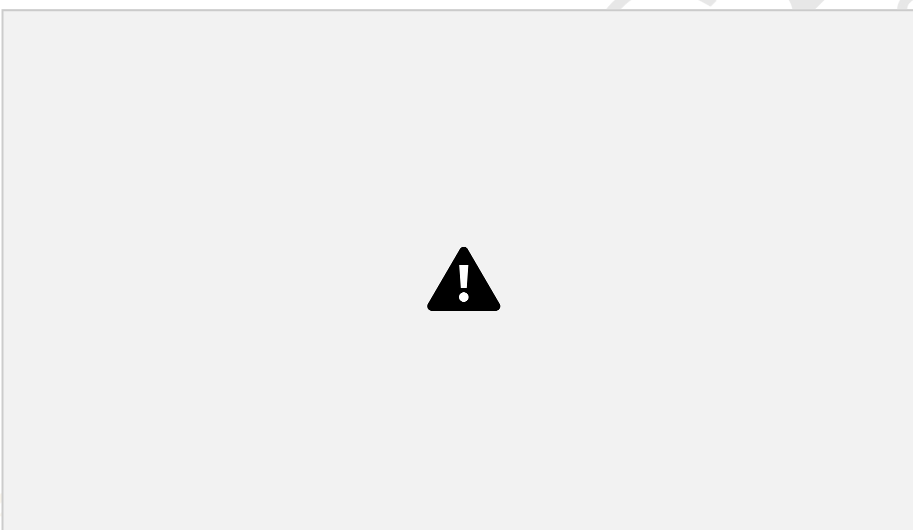
An irrational number cannot be written as a fraction p/q for any integers p and q. Its decimal expansion is non-terminating and non-repeating it goes on forever with no pattern. Symbol: ℚ′ or Q̄. Examples: √2, π, √3, e.

Real Numbers

Real numbers form the complete number system they include every rational number and every irrational number. Every point on the number line corresponds to exactly one real number. Symbol: ℝ.

Number System Revision Notes

The structural diagram below shows how each type of number fits inside the larger categories like a set of nested boxes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Classification of Numbers

The diagram above shows the hierarchy clearly. Natural numbers sit deepest inside the structure. Every natural number is also a whole number. Every whole number is also an integer. Every integer is also a rational number. Rational numbers and irrational numbers together form the complete set of real numbers.

Relationship Between Different Types of Numbers

The key relationship to memorise: ℕ ⊂ W ⊂ ℤ ⊂ ℚ ⊂ ℝ. The symbol ⊂ means "is a subset of." Irrational numbers sit alongside rational numbers within real numbers but share no overlap with them a number is either rational or irrational, never both.

Rational Numbers Revision Notes

Definition of Rational Numbers

A rational number is any number that can be expressed in the form p/q, where p and q are integers and q ≠ 0. The word rational comes from ratio it is a number that can be expressed as a ratio of two integers.

Properties of Rational Numbers

Rational numbers are closed under addition, subtraction, multiplication, and division (except dividing by zero). The sum, difference, and product of any two rational numbers is always another rational number. Between any two rational numbers, there are infinitely many other rational numbers this is called the density property.

Examples of Rational Numbers

- Integers: −5 = −5/1
- Fractions: 3/7, −2/9
- Terminating decimals: 0.25 = 1/4
- Repeating decimals: 0.666… = 2/3
- Zero: 0 = 0/1

Irrational Numbers Revision Notes

Definition of Irrational Numbers

An irrational number is a real number that cannot be written as a fraction p/q for any integers p and q. It is impossible to express it exactly as a ratio of two whole numbers. The word irrational means "not a ratio."

Examples of Irrational Numbers

- √2, √3, √5, √7 (square roots of non-perfect squares)
- π (pi) the ratio of a circle's circumference to its diameter
- e (Euler's number) the base of natural logarithms
- φ (golden ratio) = (1 + √5)/2

Note: √4 = 2 is rational. √9 = 3 is rational. Only square roots of non-perfect squares are irrational.

Decimal Representation of Irrational Numbers

The decimal expansion of an irrational number never ends and never repeats. This is the simplest test: if you cannot find a repeating block in the decimal, the number is irrational.

- $\sqrt{2}$ = 1.41421356237… (never ends, no repeating block)
- π = 3.14159265358… (never ends, no repeating block)
- Compare to 1/3 = 0.333… (repeats so it is rational)

Real Numbers Revision Notes

Definition of Real Numbers

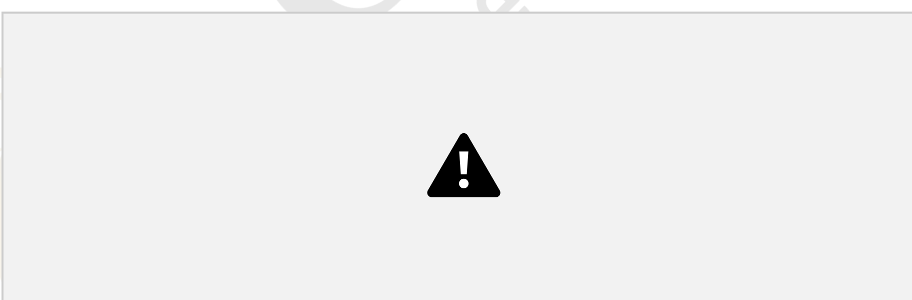
Real numbers are the complete collection of all numbers that can be placed on a number line. The formal definition: $\mathbb{R} = \mathbb{Q} \cup \mathbb{Q}'$, meaning real numbers are the union of rational and irrational numbers. Every number you encounter in Class 9 Maths is a real number.

Representation of Real Numbers

Real numbers are represented on a straight, endless line called the real number line (or simply the number line). Every real number corresponds to exactly one point, and every point corresponds to exactly one real number. There are no gaps the line is completely filled.

Real Numbers on the Number Line

The diagram below shows rational and irrational numbers placed on the same number line.



Important Concepts from the Chapter

Rational and Irrational Numbers Together Form Real Numbers

This is one of the most commonly tested ideas in Chapter 3. Students must clearly remember that the real number set R is made up of two non-overlapping groups:

rational numbers (Q) and irrational numbers. There is no
number that belongs to both groups at the same time. This can be written as:

R = Q ∪ (Irrational Numbers)

Q ∩ Irrational = Empty Set (no overlap)

In plain words:

Real Numbers = Rational Numbers + Irrational Numbers

Number Line Representation

Students are often asked to locate irrational numbers like $\sqrt{2}$ or $\sqrt{3}$ on a number line. This is done using the Pythagoras theorem. For example, to locate $\sqrt{2}$, draw a right-angled triangle with both legs of length 1 unit. The hypotenuse of that triangle will have a length of $\sqrt{2}$ units. Transfer this length to the number line using a compass to mark the exact position.

This method shows that even irrational numbers have precise, definite positions on the number line they are not vague or approximate. This is a key insight from the chapter.

Decimal Expansion of Numbers

Understanding decimal expansions is a commonly tested concept in board exams. Here is a quick summary of how to identify the type of number from its decimal form:

• Decimal ends after a few digits: the number is rational (e.g., 0.75 = 3/4)

• Decimal has a block that keeps repeating: the number is rational (e.g., 0.666... = 2/3)

• Decimal goes on forever with no repeating block: the number is irrational (e.g., π, $\sqrt{2}$)

Properties of Real Numbers

Closure Property

The closure property states that when you perform an operation on two real numbers, the result is also a real number. For real numbers a and b:

• $a + b$ is a real number

• $a - b$ is a real number

• $a \times b$ is a real number

• $a \div b$ is a real number, as long as $b \neq 0$

Commutative Property

The commutative property applies to addition and multiplication.
It means the order of the two numbers does not change the result.

- $a + b = b + a$ (Example: $3 + 5 = 5 + 3 = 8$)
- $a \times b = b \times a$ (Example: $4 \times 6 = 6 \times 4 = 24$)

Note: Subtraction and division are not commutative. For instance, $7 - 3 \neq 3 - 7$.

Associative Property

The associative property applies to addition and multiplication. It means the way numbers are grouped in brackets does not change the result.

- $(a + b) + c = a + (b + c)$ (Example: $(2 + 3) + 4 = 2 + (3 + 4) = 9$)
- $(a \times b) \times c = a \times (b \times c)$ (Example: $(2 \times 3) \times 4 = 2 \times (3 \times 4) = 24$)

Distributive Property

The distributive property connects multiplication with addition or subtraction. It is very useful for simplifying expressions.

- $a \times (b + c) = a \times b + a \times c$
- $a \times (b - c) = a \times b - a \times c$

Example: $3 \times (4 + 5) = 3 \times 4 + 3 \times 5 = 12 + 15 = 27$

Important Rules to Remember

Identifying Rational Numbers

A number is rational if any one of the following is true:

- It can be written as p/q where p and q are integers and $q \neq 0$
- Its decimal expansion terminates (e.g., 0.5, 1.25, 3.75)
- Its decimal expansion is non-terminating but repeating (e.g., 0.333..., 0.142857142857...)
- It is a whole number, a negative integer, or zero

Identifying Irrational Numbers

A number is irrational if:

- It cannot be written as p/q for any pair of integers p and q
- Its decimal expansion is non-terminating and non-repeating

• It is the square root of a non-perfect square (√2, √3, √5, √7, √10, etc.)

• It is a known constant like π or Euler's number e

Quick Check: √9 = 3 (rational), √4 = 2 (rational), but √2 ≈ 1.414... (irrational). Always check if the square root simplifies to a whole number first.

Understanding Decimal Expansions

TYPE OF DECIMAL → TYPE OF NUMBER

Terminating → Rational

e.g. 0.5, 1.25, 3.0

Non-terminating → Rational

Repeating e.g. 0.333..., 0.142857...

Non-terminating → Irrational

Non-Repeating e.g. √2, √3, π

Number System Summary Table

Comparison of Different Types of Numbers

Type	Symbol	Includes Zero?	Includes Negatives?	Example
Natural	N	No	No	1, 2, 3, 100
Whole	W	Yes	No	0, 1, 2, 50
Integers	Z	Yes	Yes	-5, 0, 4
Rational	Q	Yes	Yes	1/2, -3, 0.75
Irrational	—	No	Yes (e.g. -√2)	√2, π, √3

Real	R	Yes	Yes	All of the above

Solved Examples for Quick Revision

Example on Rational Numbers

Example 1:

Question: Is 0.6 repeating (i.e., 0.666...) a rational number?

Solution:

Let $x = 0.666...$

Then $10x = 6.666...$

Subtracting: $10x - x = 6.666... - 0.666...$

$9x = 6$

$x = 6/9 = 2/3$

Since $2/3$ is in p/q form with integers $p = 2$, $q = 3$, and $q \neq 0$,

the number 0.666... is a rational number.

Example 2:

Question: Express 1.75 as a rational number in p/q form.

Solution:

$1.75 = 175/100 = 7/4$

Here $p = 7$, $q = 4$, both are integers, and $q \neq 0$.

So 1.75 is a rational number. Its decimal terminates, confirming this.

Example on Irrational Numbers

Example 3:

Question: Show that $\sqrt{5}$ is irrational.

Solution (by contradiction):

Assume $\sqrt{5}$ is rational. Then $\sqrt{5} = p/q$ in lowest terms.

So $5 = p^2/q^2$, meaning $p^2 = 5q^2$.

This means 5 divides p^2, so 5 divides p. Let p = 5m.

Then $(5m)^2 = 5q^2 \rightarrow 25m^2 = 5q^2 \rightarrow q^2 = 5m^2$.

This means 5 divides q^2 and therefore 5 divides q.

But then 5 divides both p and q contradicting the assumption

that p/q is in lowest terms. So $\sqrt{5}$ cannot be rational.

Therefore $\sqrt{5}$ is irrational.

Example 4:

Question: Is $\pi/2$ rational or irrational?

Solution:

π is irrational. Dividing an irrational number by a non-zero

rational number gives an irrational number.

2 is rational and non-zero.

Therefore $\pi/2$ is irrational.

Example on Real Numbers

Example 5:

Question: Classify each of the following as rational or irrational:

(a) $\sqrt{16}$ (b) $\sqrt{12}$ (c) 3.14 (d) 0.010010001... (e) -7

Solution:

(a) $\sqrt{16} = 4 \rightarrow$ rational (perfect square)

(b) $\sqrt{12} = 2\sqrt{3} \rightarrow$ irrational ($\sqrt{3}$ is irrational)

(c) $3.14 = 314/100 = 157/50 \rightarrow$ rational (terminating decimal)

(d) 0.010010001... $\rightarrow$ irrational (non-terminating, non-repeating)

(e) $-7 = -7/1 \rightarrow$ rational (integer is a rational number)