Class 9 Maths Chapter 4 Exploring Algebraic Identities Notes

Class 9 Maths Chapter 4 Exploring Algebraic Identities Notes Free PDF Download is prepared based on the latest CBSE and NCERT syllabus. These notes will help in school exams, board exams, and quick revision. They help students understand the chapter clearly, revise faster, and prepare for exams with confidence.

Introduction to Algebraic Identities

What Are Algebraic Identities?

An algebraic identity is an equation that is true for every value of the variables in it. For example, $(a + b)^2 = a^2 + 2ab + b^2$ is an identity because no matter what values you choose for a and b, both sides always give the same result. This is different from an equation like $x + 3 = 7$, which is only true when $x = 4$.

Key Terms to Remember

- **Algebraic Expression:** A combination of variables, constants, and arithmetic operations. Example: $3x^2 + 5x - 7$.
- **Variable:** A letter that represents an unknown or changing value. Common variables: a, b, x, y.
- **Constant:** A fixed number in an expression that does not change. In $3x + 7$, the number 7 is a constant.
- **Coefficient:** The numerical factor of a variable term. In $5x^2$, the coefficient is 5.
- **Identity:** An equation that is true for all values of the variable. Both sides are equivalent expressions, not just equal for specific values.

Important Algebraic Identities

The three core algebraic identities for Class 9 are shown in the formula chart below. These form the foundation of all expansion and factorisation work in the chapter.

Square of a Sum

$(a + b)^2 = a^2 + 2ab + b^2$

The square of the sum of two terms equals the square of the first term, plus twice the product of both terms, plus the square of the second term.

Example: $(7 + 3)^2$

$= 7^2 + 2(7)(3) + 3^2$

$= 49 + 42 + 9 = 100$.

Check: $10^2 = 100$

Square of a Difference

$(a - b)^2 = a^2 - 2ab + b^2$

The square of the difference of two terms equals the square of the first, minus twice the product of both, plus the square of the second. Note: the last term b^2 is always positive.

Example: $(8 - 3)^2$

$= 8^2 - 2(8)(3) + 3^2$

$= 64 - 48 + 9 = 25$.

Check: $5^2 = 25$

Difference of Squares

$a^2 - b^2 = (a + b)(a - b)$

The difference of two perfect squares always factors into the product of the sum and the difference of the two terms.

Example: $9^2 - 4^2$

$= (9 + 4)(9 - 4)$

$= 13 \times 5 = 65$.

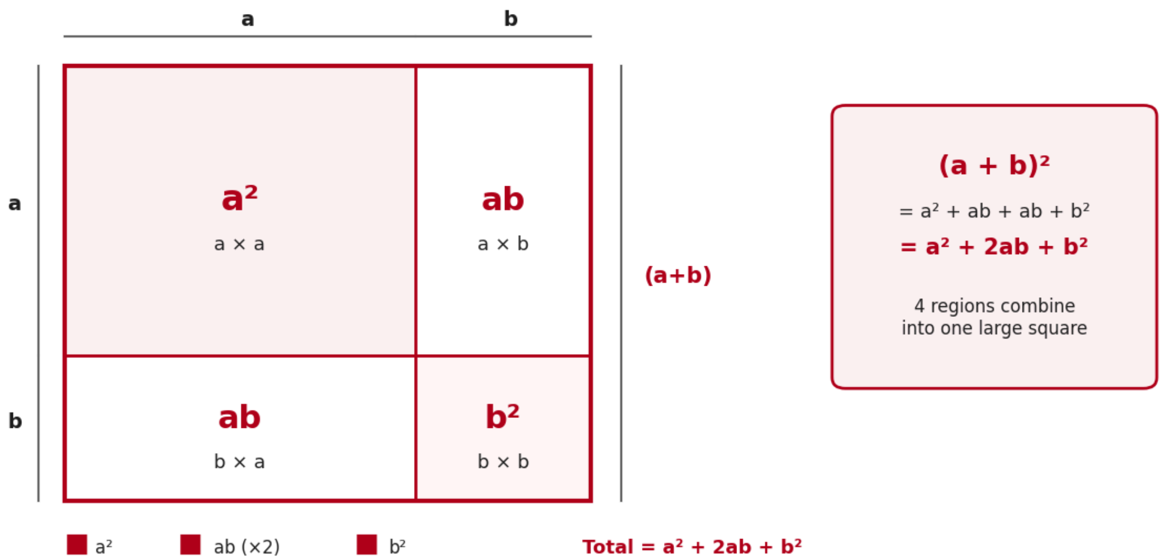
Check: $81 - 16 = 65$

Visual Understanding of Algebraic Identities

Geometric Interpretation of $(a + b)^2$

The best way to understand $(a + b)^2 = a^2 + 2ab + b^2$ is to think of it as the area of a large square with side $(a + b)$. Divide that square into four smaller regions. You get one $a \times a$ square, two $a \times b$ rectangles, and one $b \times b$ square. Add the areas: $a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$.

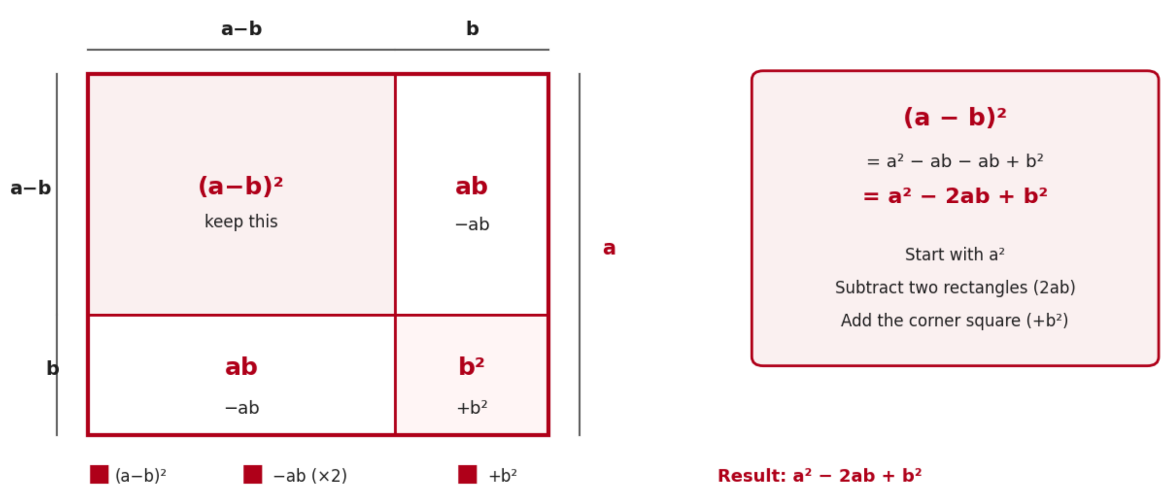
Visual Proof of $(a + b)^2 = a^2 + 2ab + b^2$



Geometric Interpretation of $(a - b)^2$

For $(a - b)^2$, imagine a square of side a. You want to remove a strip of width b from two sides to get a square of side $(a - b)$. When you remove the corner regions, you subtract two rectangles of area ab but add back the corner b^2 that was subtracted twice. This gives: $a^2 - ab - ab + b^2 = a^2 - 2ab + b^2$.

Visual Proof of $(a - b)^2 = a^2 - 2ab + b^2$



Algebraic Identities Formula Chart

The chart below summarises all three identities together ideal for quick revision before exams.

Important Algebraic Identities

Square of a Sum	Square of a Difference	Difference of Squares
$(a + b)^2 = a^2 + 2ab + b^2$	$(a - b)^2 = a^2 - 2ab + b^2$	$a^2 - b^2 = (a + b)(a - b)$
Middle term = +2ab Both signs are positive	Middle term = −2ab Last term remains $+b^2$	Two factors: sum × difference Used for factorisation
Example $(5 + 3)^2$ $= 25 + 30 + 9$ $= 64$	**Example** $(9 - 2)^2$ $= 81 - 36 + 4$ $= 49$	**Example** $25 - 9$ $= (5 + 3)(5 - 3)$ $= 8 \times 2 = 16$

Applications of Algebraic Identities

Simplifying Expressions

Identities let you replace a longer multiplication with a direct formula. Instead of expanding $(x + 4)(x + 4)$ step by step, you recognise it as $(x + 4)^2$ and directly write $x^2 + 8x + 16$. This saves time and reduces errors.

Faster Calculations

Identities work brilliantly for mental arithmetic. To calculate 99^2, write it as $(100 - 1)^2 = 10000 - 200 + 1 = 9801$. To calculate 51×49, recognise it as $(50 + 1)(50 - 1) = 50^2 - 1 = 2500 - 1 = 2499$. These are standard exam uses of identities.

Expanding Algebraic Expressions

When an expression is written in factored form and you need its expanded form, apply the matching identity. For example, $(2x + 3y)^2$ expands directly as $4x^2 + 12xy + 9y^2$ using the $(a + b)^2$ identity with $a = 2x$ and $b = 3y$.

Expansion Using Algebraic Identities

Expanding Binomials

Identify which identity fits the form of the expression. Replace the variables a and b with the actual terms, then substitute into the formula.

Examples:

- $(x + 5)^2 = x^2 + 10x + 25$ (use $a = x$, $b = 5$)

- $(3a - 2b)^2 = 9a^2 - 12ab + 4b^2$ (use $a = 3a$, $b = 2b$)
- $(4x + 3y)(4x - 3y) = 16x^2 - 9y^2$ (use $a = 4x$, $b = 3y$)

Simplifying Expressions Using Identities

If you see a pattern matching an identity, do not multiply term by term apply the formula directly. This avoids sign errors and saves steps, especially in exam conditions.

Factorisation Using Algebraic Identities

Factoring Difference of Squares

Any expression of the form $a^2 - b^2$ can be factored as $(a + b)(a - b)$. The key is recognising two perfect square terms with a minus sign between them.

Examples:

- $x^2 - 16 = x^2 - 4^2 = (x + 4)(x - 4)$
- $9y^2 - 25 = (3y)^2 - 5^2 = (3y + 5)(3y - 5)$
- $4a^2 - 49b^2 = (2a)^2 - (7b)^2 = (2a + 7b)(2a - 7b)$

Recognising Common Patterns

Before factorising, always check: are both terms perfect squares? Is there a subtraction sign between them? If yes, the difference of squares identity applies directly. Similarly, a perfect square trinomial (three terms with $+2ab$ in the middle) can be compressed back into $(a + b)^2$.

Example: $x^2 + 6x + 9 = x^2 + 2(x)(3) + 3^2 = (x + 3)^2$

Important Concepts from the Chapter

Relationship Between Expansion and Factorisation

Expansion and factorisation are reverse operations. Expanding takes a factored form like $(a + b)^2$ and opens it to $a^2 + 2ab + b^2$. Factorisation takes the expanded form and compresses it back. Knowing one identity gives you the tool for both directions.

Identifying the Correct Identity

Before solving, identify which of the three identities matches the structure of your expression. Check: is it a sum squared, a difference squared, or a difference of two squares? The structure tells you which formula to apply.

Pattern Recognition in Algebra

The skill this chapter builds is pattern recognition seeing that $(2x + 5)^2$ matches the $(a + b)^2$ structure, or that $49m^2 - 36n^2$ is a difference of squares. This ability becomes the foundation for all algebraic manipulation in higher classes.

Common Mistakes Students Make

Sign Errors in Expansions

The most common error is writing $(a - b)^2 = a^2 - 2ab - b^2$. The last term b^2 is always positive, because $(-b) \times (-b) = +b^2$. Never write it as minus b^2.

Similarly, $(a + b)^2$ is never $a^2 + b^2$. The middle term 2ab is always there and is always forgotten by students under exam pressure.

Incorrect Application of Identities

Students sometimes apply $(a + b)^2$ when the expression is actually $(a + b)(a - b)$. Always check the signs before choosing the identity. If both factors are identical, it is a square. If one has a plus and one has a minus, it is the difference of squares.

Errors While Factorising Expressions

When factorising using $a^2 - b^2 = (a + b)(a - b)$, students sometimes forget to take the square root of the terms. For $9x^2$, the value of a is 3x, not 9x. Always extract the square root correctly before writing the factors.

Solved Examples for Quick Revision

Example Using $(a + b)^2$

Expand $(3x + 4y)^2$.

Solution: Use $(a + b)^2 = a^2 + 2ab + b^2$ with $a = 3x$, $b = 4y$.

$= (3x)^2 + 2(3x)(4y) + (4y)^2 = 9x^2 + 24xy + 16y^2$

Example Using $(a - b)^2$

Evaluate 97^2 using an identity.

Solution: Write $97 = 100 - 3$. Apply $(a - b)^2$ with $a = 100$, $b = 3$.

$= 100^2 - 2(100)(3) + 3^2 = 10000 - 600 + 9 = 9409$

Example Using $a^2 - b^2$

Factorise $4m^2 - 81n^2$.

Solution: Write as $(2m)^2 - (9n)^2$. Apply $a^2 - b^2 = (a + b)(a - b)$ with $a = 2m$, $b = 9n$.

$= (2m + 9n)(2m - 9n)$