Class 9 Maths Chapter 7 'The Mathematics of Maybe: Introduction to Probability ' Notes

Probability is a numerical measure of how likely an event is to happen. It is always a number between 0 and 1 (or 0% to 100%), where 0 means the event is impossible and 1 means it is certain.

Key Definitions in Probability

Term	Definition	Example
Random Experiment	An activity whose outcome cannot be predicted with certainty, but all possible outcomes are known.	Tossing a coin, rolling a die
Trial	Each single performance of a random experiment.	Tossing a coin once
Outcome	The result obtained from a single trial of a random experiment.	Getting **Heads**
Sample Space (S)	The set of all possible outcomes of a random experiment.	S={H, T}) for a coin toss
Event (E)	One or more outcomes from the sample space that satisfy a given condition.	E = Heads
Favourable Outcomes	The outcomes in the sample space that satisfy the event's condition.	{H}favours the event "getting Heads"
Equally Likely Outcomes	Outcomes that have the same chance of occurring.	Each face of a fair die

Sure (Certain) Event	An event that always occurs; its probability is 1.	Rolling a number ≤ 6 on a standard die
Impossible Event	An event that can never occur; its probability is 0.	Rolling a 7 on a standard die
Complementary Event ($\bar{E}$) or (E')	The event that (E) does **not** occur. For any event, $P(E)+P(\bar{E})=1$.	If E = Heads, then $\bar{E}$ = Tails

- **The Probability Scale**

Probability is expressed as a number from 0 to 1.

$\mathbf{0 \leq P(E) \leq 1}$

The probability of any event is always between 0 (impossible) and 1 (certain), inclusive.

- **Subjective vs. Objective Thinking**
  Sometimes people make predictions based on personal feelings like ‘It looks dark outside, so it will probably rain’.This is called a subjective judgment. But when predictions are made using actual data, patterns, or mathematical reasoning, they are objective. In mathematics, probability is always objective.

Sample Space and Events

The sample space (S) is the complete list of all possible outcomes of a random experiment, written as a set.

🪙1 Coin Toss

- S = {H, T}
- Number of outcomes = 2

🪙🪙 2 Coins Tossed

- S = {HH, HT, TH, TT}
- Number of outcomes = 4

🎲 1 Die Rolled

- S = {1, 2, 3, 4, 5, 6}
- Number of outcomes = 6

🎲🎲 2 Dice Rolled

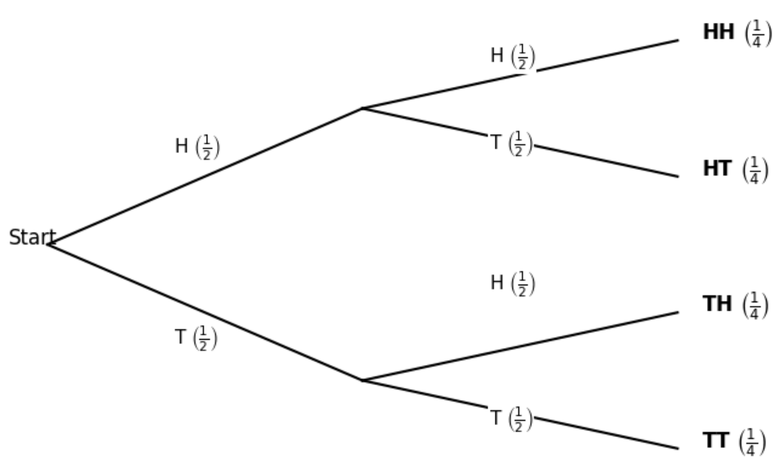
- S = {(1,1), (1,2), … (6,6)}
- Number of outcomes = 36

What is an Event?
An event is a subset of the sample space. It is the specific outcome or group of outcomes we are looking for. For example, if we roll a die and define Event E as 'getting an even number',then E = {2, 4, 6}.

Visualising All Possible Outcomes: Tree Diagrams

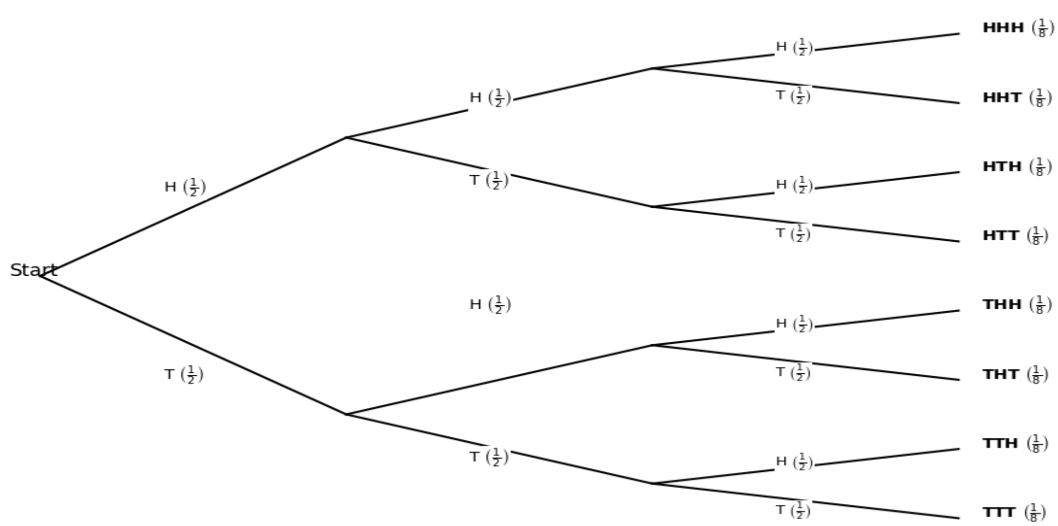
A tree diagram is a visual tool used to systematically list all possible outcomes of a multi-step random experiment. Each branch represents one possible outcome at that stage. By following every path from start to finish, you get the complete sample space.

Tree Diagram: Tossing a Coin Twice



Total outcomes = 2 × 2 = 4. Each outcome has probability ½ × ½ = ¼.
This confirms that tossing 2 coins gives 4 equally likely outcomes.

Tree Diagram: Tossing a Coin Three Times



When a coin is tossed three times, we get 2 × 2 × 2 = 8 outcomes.
Sample Space: S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
Total = 8 outcomes
Each has probability = ½ × ½ × ½ = ⅛

If you have n steps, each with k equally likely outcomes, the total number of outcomes = k^n. For 3 coin tosses: $2^3 = 8$. For 2 dice: $6^2 = 36$.

Empirical Probability: Formula and Examples

Experimental probability, also called empirical probability, is calculated by actually performing an experiment and recording what happens.

Formula:
P(E) = Number of times event E occurs ÷ Total number of trials

Example:
A bag of 1000 tomatoes is inspected. 40 are found to be rotten. What is the probability that a randomly picked tomato is (i) rotten (ii) good?
Solution: Total tomatoes (n) = 1000
Number of rotten tomatoes = 40
Number of good tomatoes = 1000 − 40 = 960

(i) P(rotten) = 40/1000 = 1/25 = 0.04
(ii) P(good) = 960/1000 = 24/25 = 0.96
P(rotten) + P(good) = 0.04 + 0.96 = 1

Theoretical Probability: Formula and Examples

Theoretical probability is calculated without performing any experiment. It uses mathematical reasoning and assumes that all outcomes in the sample space are equally likely.

Formula:
P(E) = Number of favourable outcomes ÷ Total number of possible outcomes
This formula works only when all outcomes are equally likely (fair coin, unbiased die, well-shuffled cards).

Key Properties of Theoretical Probability

- P(impossible event) = 0 (e.g., rolling 7 on a standard die)
- P(sure/certain event) = 1 (e.g., rolling a number ≤ 6 on a die)
- $0 \leq P(E) \leq 1$ for any event E
- Sum of probabilities of all outcomes = 1
- $P(\bar{E}) = 1 - P(E)$, where $\bar{E}$ is the complement of E

Example: Two fair coins are tossed simultaneously. Find the probability of getting:
(i) exactly 2 Heads
(ii) exactly 1 Head
(iii) at least 1 Head
(iv) no Heads.
Solution: Sample Space: S = {HH, HT, TH, TT}
Total outcomes = 4
(i) Exactly 2 Heads: {HH}, P = 1/4
(ii) Exactly 1 Head: {HT, TH}, P = 2/4 = 1/2
(iii) At least 1 Head: {HH, HT, TH}, P = 3/4
(iv) No Heads (2 Tails): {TT}, P = ¼

Example: Two dice are thrown simultaneously. Find the probability that:
(i) the sum is 7
(ii) the sum is 8
(iii) both show the same number
(iv) sum > 10.
Solution: Total outcomes = $6 \times 6 = 36$
(i) Sum = 7: (1,6), (2,5), (3,4), (4,3), (5,2),(6,1); P = 6/36 = 1/6
(ii) Sum = 8: (2,6), (3,5), (4,4), (5,3), (6,2); P = 5/36
(iii) Both same: (1,1), (2,2), (3,3), (4,4), (5,5), (6,6); P = 6/36 = 1/6
(iv) Sum > 10: (5,6), (6,5), (6,6); P = 3/36 = 1/12

Example: A card is drawn from a well-shuffled deck of 52 cards. Find the probability of drawing:
(i) a Queen
(ii) a red card
(iii) a black King
(iv) a face card
(v) neither a Jack nor a King.
Solution: Total outcomes = 52
(i) Queens = 4 (one per suit); P = 4/52 = 1/13
(ii) Red cards = 26 (♥ + ♦); P = 26/52 = 1/2
(iii) Black Kings = 2 (K♠ and K♣); P = 2/52 = 1/26
(iv) Face cards = 12 (4J + 4Q + 4K); P = 12/52 = 3/13
(v) Jacks = 4, Kings = 4, total J or K = 8.
Remaining = 52 − 8 = 44; P = 44/52 = 11/13

Complementary Events: Formula and Examples

The complement of an event E, written as $\bar{E}$ (read as "E bar"), is the event that E does NOT happen. Together, E and $\bar{E}$ cover all possibilities in the sample space.

$$P(E) + P(\bar{E}) = 1 \Rightarrow P(\bar{E}) = 1 - P(E)$$

Example: In a class of 30 students, 6 are class monitors. If one student is selected at random, find:

(i) P(selected is a monitor)

(ii) P(selected is NOT a monitor).

Solution: Total students = 30, Monitors = 6

(i) P(monitor) = 6/30 = 1/5 = 0.2

(ii) P(not a monitor) = 1 − 1/5 = 4/5 = 0.8