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HOTS Questions on Chapter 11 Class 10 Areas Related To Circles

HOTS Questions Involving Logical Reasoning

Question 15:

Two students solve this problem: "Find the area of a segment with $r = 14$ cm and $\theta = 90°$."

Student A: Area = $(1/4) \times \pi r^2 - (1/2) \times r^2$ Student B: Area = $(\theta/360) \times \pi r^2 - (1/2) \times b \times h$

Both get correct numerical answers. Whose approach is more generalizable? Why?

Answer:

Student A's method:

$= (1/4) \times \pi \times 196 - (1/2) \times 196$

$= 154 - 98 = 56$ cm²

Works because $\theta = 90°$ makes the

triangle a right triangle (base = height = r).

This is a SPECIAL CASE approach.

Student B's method:

$= (\theta/360) \times \pi r^2 - (1/2) \times r^2 \times \sin\theta$

$= (90/360) \times \pi \times 196 - (1/2) \times 196 \times \sin 90°$

$= 154 - (1/2) \times 196 \times 1$

$= 154 - 98 = 56$ cm²

Student B's approach is MORE GENERALIZABLE

because it uses $\sin\theta$ which works for

ANY angle, not just 90°.

For $\theta = 60°$: $(1/2) \times r^2 \times \sin 60° = (\sqrt{3}/4)r^2$

For $\theta = 120°$: $(1/2) \times r^2 \times \sin 120° = (\sqrt{3}/4)r^2$

Student A's shortcut only works at $\theta = 90°$.

Student B's formula works for all angles.

Answer: Student B's approach is more generalizable.

Question 16: A student solved: "Find the area of major segment if r = 7 cm and θ = 60°."

Student's solution:

Area of minor sector = $(60/360) \times (22/7) \times 49 = 25.67$ cm²

Area of major segment = Area of circle - Minor sector

$= 154 - 25.67 = 128.33$ cm²

Find the error and give the correct solution.

Answer:

The ERROR is in the last step.

The student subtracted minor SECTOR from the circle,

but major segment ≠ circle - minor sector.

Correct relationship:

Major segment = Circle area - Minor segment

Minor segment = Minor sector - Triangle

(Student forgot to subtract the triangle area!)

Correct solution:

Minor sector area = $(60/360) \times (22/7) \times 49$

$= 25.67$ cm²

Triangle OAB ($\theta = 60°$, equilateral triangle):

$= (\sqrt{3}/4) \times r^2 = (\sqrt{3}/4) \times 49$

$= 12.25\sqrt{3} = 21.22$ cm²

Minor segment = 25.67 - 21.22 = 4.45 cm²

Circle area = (22/7) × 49 = 154 cm²

Major segment = 154 - 4.45 = 149.55 cm²

(Not 128.33 as the student calculated!)

Correct Answer: Major segment = 149.55 cm²

HOTS Question 17: Find the area of a semicircle of radius 7 cm using: Method 1: Sector formula Method 2: Half of circle formula Compare and explain why both give the same result.

Answer:

Method 1: Sector formula

$\theta = 180°$ for semicircle

Area $= (\theta/360) \times \pi r^2$

= (180/360) × (22/7) × 49

= (1/2) × 154

= 77 cm²

Method 2: Half of circle

Area $= (1/2) \times \pi r^2$

= (1/2) × (22/7) × 49

= 77 cm²

Both methods give 77 cm²

Explanation:

A semicircle IS a sector with $\theta = 180°$.

$\theta/360 = 180/360 = 1/2$

So $(\theta/360) \times \pi r^2 = (1/2) \times \pi r^2$

The sector formula is the GENERAL form.

Half circle formula is a SPECIAL CASE

when $\theta = 180°$. They're identical!

Case-Based HOTS Questions on Areas Related to Circles

Question 18 (Case Study):

A school playground is circular with a radius of 35 m. The school authority decided to divide it into three equal sectors. Sector 1 will have a garden, Sector 2 will be a sports area, and Sector 3 will be an open space.

Answer the following: a) What is the central angle of each sector? b) Find the area of each sector. c) The garden sector will be fenced along its boundary. Find the fencing length needed. d) If fencing costs ₹25 per meter, find the total cost.

Answer:

a) Central angle of each sector:

Total angle = 360°

Each sector = 360°/3 = 120°

b) Area of each sector:

$= (120/360) \times (22/7) \times 35^2$

$= (1/3) \times (22/7) \times 1225$

$= (1/3) \times 3850$

$= 1283.33\ m^2$

c) Fencing boundary of garden sector includes:

- Two radii: 2 × 35 = 70 m

- Arc length: (120/360) × 2 × (22/7) × 35

= (1/3) × 2 × (22/7) × 35

= (1/3) × 220

= 73.33 m

Total fencing = 70 + 73.33 = 143.33 m

d) Cost = 143.33 × 25 = ₹3583.25

Answers: a) Each sector = 120° b) Each area = 1283.33 m^2 c) Fencing needed = 143.33 m d) Cost = ₹3583.25

Question 19 (Case Study):

A pie chart represents a school's land usage. The circle has radius 10.5 cm. The central angles for different uses are:

- Classrooms: 120°
- Playground: 90°
- Gardens: 60°
- Parking: 45°
- Others: 45°

a) Verify that all angles account for the complete circle. b) Find the area representing playground. c) Which land use has the largest area on the pie chart? d) Find the area of gardens + parking combined.

Answer Explanation:

a) Verification:

120 + 90 + 60 + 45 + 45 = 360°

All angles account for complete circle.

b) Playground area (θ = 90°):

= (90/360) × (22/7) × 10.5^2

= (1/4) × (22/7) × 110.25

= (1/4) × 346.5

= 86.625 cm^2

c) Largest angle = 120° (classrooms)

Largest area = Classrooms sector

Area = (120/360) × (22/7) × 110.25

= (1/3) × 346.5

= 115.5 cm^2

d) Gardens (60°) + Parking (45°) = 105°

Combined area = (105/360) × (22/7) × 110.25

= (7/24) × 346.5

= 101.04 cm^2

Answers: a) Verified b) Playground = 86.625 cm^2 c) Classrooms (120°) has largest area d) Gardens + Parking = 101.04 cm^2

Question 20 (Case Study):

A farmer has a circular field of radius 28 m. He divides it into two segments using a chord that subtends 90° at the center. He plants wheat in the minor segment and vegetables in the major segment.

a) Find the area for wheat cultivation. b) Find the area for vegetables. c) If wheat yields ₹800 per m^2 and vegetables yield ₹600 per m^2, find the total revenue. d) If the farmer wants to put a fence along the chord, how long should it be?

Answer Explanation:

r = 28 m, θ = 90°

a) Minor sector area:

$= (90/360) \times (22/7) \times 28^2$

$= (1/4) \times (22/7) \times 784$

$= (1/4) \times 2464$

$= 616\ m^2$

Triangle OAB (right triangle, legs = 28):

$= (1/2) \times 28 \times 28 = 392\ m^2$

Wheat area (minor segment):

$= 616 - 392 = 224\ m^2$

b) Total circle = $(22/7) \times 784 = 2464\ m^2$

Vegetable area (major segment):

$= 2464 - 224 = 2240\ m^2$

c) Revenue:

Wheat = 224 × 800 = ₹1,79,200

Vegetables = 2240 × 600 = ₹13,44,000

Total = ₹15,23,200

d) Chord length:

Since $\theta = 90°$ and both sides = r = 28:

$Chord^2 = 28^2 + 28^2 = 784 + 784 = 1568$

Chord = $\sqrt{1568} = 28\sqrt{2} = 39.6$ m

Answers: a) Wheat area = $224\ m^2$ b) Vegetable area = $2240\ m^2$ c) Total revenue = ₹15,23,200 d) Fence length = $28\sqrt{2} \approx 39.6$ m

Mixed HOTS Practice Questions on Areas Related to Circles

Question 21: Find the area of the region common to two circles of radii 5 cm each that intersect such that each passes through the center of the other.

Answer:

Distance between centers = 5 cm = r

Each circle's intersection sector:

At each center, the chord connecting the two

intersection points subtends $60° \times 2 = 120°$

Area of one sector ($r = 5$, $\theta = 120°$):

$= (120/360) \times \pi \times 25$

$= (1/3) \times (22/7) \times 25$

$= 550/21$

$= 26.19\ cm^2$

Area of equilateral triangle (side = 5):

$= (\sqrt{3}/4) \times 25$

$= 6.25\sqrt{3}$

$= 10.83\ cm^2$

Area of one segment = $26.19 - 10.83 = 15.36\ cm^2$

Common area = 2 segments = $2 \times 15.36 = 30.72\ cm^2$

Answer: Common area = $30.72\ cm^2$

Question 22: A horse is tied at the corner A of a square field ABCD of side 21 m with a rope of length 14 m. Find the area the horse can graze. If the rope is lengthened by 7 m, find the additional area the horse can now reach.

Answer:

Step 1: Original rope = 14 m

Corner of square = 90° interior angle

So horse can graze in a quadrant.

Original grazing area:

$= (90/360) \times \pi \times 14^2$

$= (1/4) \times (22/7) \times 196$

$= (1/4) \times 616$

$= 154\ m^2$

Step 2: New rope = 14 + 7 = 21 m

New grazing area:

$= (90/360) \times \pi \times 21^2$

$= (1/4) \times (22/7) \times 441$

$= (1/4) \times 1386$

$= 346.5\ m^2$

Step 3: Additional area gained:

= 346.5 - 154

$= 192.5\ m^2$

Note: The extra 7 m only operates in the

quadrant direction (90°), not beyond square sides.

Answer: Original area = $154\ m^2$, Additional area = $192.5\ m^2$

Question 23: In the figure, ABC is a right-angled triangle with AB = 6 cm, BC = 8 cm, ∠B = 90°. Semicircles are drawn on all three sides as diameters. Find the area of the two crescent (lune) shaped regions formed.

Answer:

This is the famous Hippocrates Lune problem

Step 1: Find hypotenuse AC

$AC^2 = 6^2 + 8^2 = 36 + 64 = 100$

AC = 10 cm

Step 2: Area of triangle ABC

= (1/2) × 6 × 8 = 24 cm^2

Step 3: Semicircle areas:

On AB (d=6, r=3): (1/2) × π × 9 = 4.5π cm^2

On BC (d=8, r=4): (1/2) × π × 16 = 8π cm^2

On AC (d=10, r=5): (1/2) × π × 25 = 12.5π cm^2

Step 4: By Pythagoras theorem of areas:

Semicircle on AB + Semicircle on BC

= 4.5π + 8π = 12.5π

= Semicircle on AC

Step 5: Area of two lunes:

= (Semicircle AB + Semicircle BC) - Semicircle AC + Triangle

= 12.5π - 12.5π + 24

= 24 cm^2

Amazing result: Area of two lunes = Area of triangle!

Answer: Area of two lunes = 24 cm^2 (equals triangle area!)

Question 24: Four circles each of radius 7 cm are arranged so that each circle touches two others and they all enclose a small square region in the middle. Find the area of the enclosed square region.

Answer:

Each circle has r = 7 cm.

Centers form a square with side = 2r = 14 cm.

Step 1: Area of the large square formed by centers

= $14^2 = 196$ cm²

Step 2: Four quarter-circles at corners of this square

(Each quarter circle belongs to one of the 4 circles)

Total area of 4 quarter circles

$= 4 \times (1/4) \times \pi \times r^2$

$= \pi \times 49$

= 154 cm²

Step 3: Enclosed region (small central area):

= Area of center square - 4 quarter circles

= 196 - 154

= 42 cm²

Answer: Enclosed square region = 42 cm²

Question 25: The minute hand of a clock is 14 cm long. Find: a) The distance covered by the tip in 45 minutes b) The area swept by the minute hand in 20 minutes c) The area swept in one complete hour

Answer:

r = 14 cm

One complete revolution = 60 minutes = 360°

a) Distance in 45 minutes:

Angle = $(45/60) \times 360° = 270°$

Arc length = $(270/360) \times 2\pi \times 14$

$= (3/4) \times 2 \times (22/7) \times 14$

$= (3/4) \times 88$

= 66 cm

b) Area swept in 20 minutes:

Angle = (20/60) × 360° = 120°

Area = (120/360) × π × 14^2

= (1/3) × (22/7) × 196

= (1/3) × 616

= 205.33 cm^2

c) Area in one complete hour (360°):

= π × r^2

= (22/7) × 196

= 616 cm^2

(This is just the full circle area)

Answers: a) Distance = 66 cm b) Area in 20 min = 205.33 cm^2 c) Area in one hour = 616 cm^2