

## HOTS Questions on Chapter 12 Class 10 Surface Areas and Volumes

### HOTS Questions Involving Logical Reasoning

**Question 17:** Two students are finding the surface area of a solid which is a cylinder with a cone on top (both with same base radius  $r = 5$  cm, cylinder  $h = 12$  cm, cone  $h = 12$  cm).

Student A adds: Total SA = TSA of cylinder + TSA of cone  
Student B adds: Total SA = CSA of cylinder + CSA of cone + base circle area

Who is correct? Find the correct surface area.

**Answer:**

Student A is WRONG.

Student B is CORRECT.

Reason:

When cone sits on top of cylinder:

- The top circle of cylinder is hidden (joined to cone base)
- The bottom circle of cylinder = base of whole solid

So visible surfaces are:

- CSA of cylinder (side)
- CSA of cone (slant surface)
- Bottom circle of cylinder (base)
- NOT the top of cylinder (hidden)
- NOT the base circle of cone (hidden)

Cone slant height:

$$l = \sqrt{(5^2 + 12^2)} = \sqrt{(25 + 144)} = \sqrt{169} = 13 \text{ cm}$$

$$\text{CSA of cylinder} = 2\pi \times 5 \times 12 = 120\pi \text{ cm}^2$$

$$\text{CSA of cone} = \pi \times 5 \times 13 = 65\pi \text{ cm}^2$$

$$\text{Base circle} = \pi \times 25 = 25\pi \text{ cm}^2$$

$$\text{Total SA} = (120 + 65 + 25)\pi$$

$$= 210\pi$$

$$= 210 \times 22/7$$

$$= 660 \text{ cm}^2$$

Answer: Student B is correct, SA = 660 cm<sup>2</sup>

**Question 18: A student solved this problem: "Find the volume of a toy rocket made of a cylinder (r = 3, h = 10) with a cone on top (r = 3, h = 4)."**

Student's work:

$$\text{Volume} = \pi r^2 h + (1/3)\pi r^2 h$$

$$= \pi \times 9 \times 10 + (1/3) \times \pi \times 9 \times 4$$

$$= 90\pi + 12\pi = 102\pi \text{ cm}^3$$

$$= 102 \times 22/7 = 320.57 \text{ cm}^3$$

Is this correct? Identify any errors.

**Answer:**

The student's METHOD is correct.

The calculation is also correct.

$$\text{Volume of cylinder} = \pi r^2 h = \pi \times 9 \times 10 = 90\pi$$

$$\text{Volume of cone} = (1/3)\pi r^2 h = (1/3) \times \pi \times 9 \times 4 = 12\pi$$

$$\text{Total} = 102\pi$$

$$102 \times 22/7 = 2244/7 = 320.57 \text{ cm}^3$$

The student's solution is COMPLETELY CORRECT.

Important check:

$r = 3$  is same for both (valid composite solid)

Heights are separate (10 and 4)

Volumes ADD correctly

No errors found good work by the student!

Answer: The solution is completely correct no errors.

**Question 19: Find the volume of a frustum (truncated cone) with top radius 4 cm, bottom radius 6 cm, and height 8 cm using: Method 1: Direct frustum formula  
Method 2: Full cone minus small cone**

**Answer:**

Method 1: Direct frustum formula

$$\begin{aligned}V &= (\pi h/3)(R^2 + r^2 + Rr) \\&= (\pi \times 8/3)(36 + 16 + 24) \\&= (8\pi/3)(76) \\&= 608\pi/3 \\&= 636.95 \text{ cm}^3\end{aligned}$$

Method 2: Full cone - Small cone

First find full cone dimensions using similar triangles:

$$R = 6, r = 4, h = 8$$

By similar triangles:

$$r/R = h_1/(h_1 + h)$$

$$4/6 = h_1/(h_1 + 8)$$

$$4h_1 + 32 = 6h_1$$

$$2h_1 = 32$$

$$h_1 = 16 \text{ cm (height of small cone)}$$

Full cone height =  $16 + 8 = 24$  cm

Volume of full cone:

$$= \left(\frac{1}{3}\right)\pi \times 36 \times 24 = 288\pi \text{ cm}^3$$

Volume of small cone:

$$= \left(\frac{1}{3}\right)\pi \times 16 \times 16 = 256\pi/3 \text{ cm}^3$$

Frustum volume =  $288\pi - 256\pi/3$

$$= 864\pi/3 - 256\pi/3$$

$$= 608\pi/3$$

$$= 636.95 \text{ cm}^3$$

Both methods give the same answer

Method 1 is faster. Method 2 gives deeper understanding.

Answer: Volume =  $636.95 \text{ cm}^3$

### Case Based HOTS Questions on Surface Areas and Volumes

**Question 20: A company manufactures perfume bottles in the shape of a cylinder surmounted by a hemisphere. The total height of the bottle is 15 cm and the radius of the bottle is 3 cm.**

Answer the following: a) Find the volume of the bottle. b) Find the total surface area of the bottle. c) If perfume costs ₹2 per  $\text{cm}^3$ , find the value of perfume in the full bottle. d) If each bottle is packed in a cylindrical box of same radius and height 15 cm, what percentage of the box is unused?

**Answer:**

$$r = 3 \text{ cm, Total height} = 15 \text{ cm}$$

$$\text{Height of hemisphere} = r = 3 \text{ cm}$$

$$\text{Height of cylinder} = 15 - 3 = 12 \text{ cm}$$

a) Volume of bottle:

$$\text{Volume of cylinder} = \pi \times 9 \times 12 = 108\pi \text{ cm}^3$$

$$\text{Volume of hemisphere} = (2/3)\pi \times 27 = 18\pi \text{ cm}^3$$

$$\text{Total} = 126\pi = 126 \times 22/7 = 396 \text{ cm}^3$$

b) Total surface area:

$$\text{CSA of cylinder} = 2\pi \times 3 \times 12 = 72\pi \text{ cm}^2$$

$$\text{CSA of hemisphere} = 2\pi \times 9 = 18\pi \text{ cm}^2$$

$$\text{Base circle} = \pi \times 9 = 9\pi \text{ cm}^2$$

$$\text{Total} = 99\pi = 99 \times 22/7 = 311.14 \text{ cm}^2$$

c) Cost of perfume:

$$= 396 \times 2 = ₹792$$

d) Volume of box (cylinder,  $r=3$ ,  $h=15$ ):

$$= \pi \times 9 \times 15 = 135\pi \text{ cm}^3$$

Unused space:

$$= 135\pi - 126\pi = 9\pi \text{ cm}^3$$

Percentage unused:

$$= (9\pi/135\pi) \times 100$$

$$= (9/135) \times 100$$

$$= 6.67\%$$

Answers: a) Volume =  $396 \text{ cm}^3$  b) Surface area =  $311.14 \text{ cm}^2$  c) Cost = ₹792 d) Unused = 6.67%

**Question 21: A school is building a new water storage system. They have two options:**

Option A: One large cylindrical tank with radius 4 m and height 5 m. Option B: Eight smaller cylindrical tanks each with radius 1 m and height 5 m.

a) Find the total volume of water stored in Option A. b) Find the total volume in Option B. c) Which option stores more water? d) Find the total curved surface area for each option. e) If painting costs ₹50 per m<sup>2</sup>, which option is more expensive to paint (curved surface only)?

Answer Explanation:

a) Volume of Option A:

$$= \pi \times 16 \times 5 = 80\pi = 251.43 \text{ m}^3$$

b) Volume of Option B:

$$\text{Each tank} = \pi \times 1 \times 5 = 5\pi \text{ m}^3$$

$$8 \text{ tanks} = 40\pi = 125.71 \text{ m}^3$$

c) Option A stores more water

$$(251.43 > 125.71)$$

$$\text{Ratio} = 80\pi/40\pi = 2:1$$

Option A stores TWICE as much as Option B!

d) CSA of Option A:

$$= 2\pi \times 4 \times 5 = 40\pi = 125.71 \text{ m}^2$$

CSA of Option B (8 tanks):

$$= 8 \times 2\pi \times 1 \times 5 = 80\pi = 251.43 \text{ m}^2$$

e) Painting cost:

$$\text{Option A} = 125.71 \times 50 = ₹6285.71$$

$$\text{Option B} = 251.43 \times 50 = ₹12571.43$$

Option B is MORE expensive to paint!

Answers: a) Option A = 251.43 m<sup>3</sup> b) Option B = 125.71 m<sup>3</sup> c) Option A stores more water (twice as much) d) A = 125.71 m<sup>2</sup>, B = 251.43 m<sup>2</sup> e) Option B is more expensive to paint

**Question 22: A farmer has a field shaped like a right triangle with legs 12 m and 16 m. He digs a well of diameter 4 m and depth 14 m, and spreads the soil dug out evenly on the triangular field.**

a) Find the volume of soil dug out. b) Find the area of the triangular field. c) Find the height by which the field is raised. d) If the well is lined with bricks costing ₹25 per m<sup>2</sup>, find the cost of lining the curved inner surface.

Answer Explanation:

Well:  $r = 2$  m,  $h = 14$  m

Triangle: legs = 12 m, 16 m

a) Volume of soil from well:

$$= \pi r^2 h$$

$$= (22/7) \times 4 \times 14$$

$$= (22 \times 4 \times 14)/7$$

$$= 1232/7$$

$$= 176 \text{ m}^3$$

b) Area of triangular field:

$$= (1/2) \times 12 \times 16$$

$$= 96 \text{ m}^2$$

c) Rise in field level:

Volume of soil = Area of field  $\times$  Rise

$$176 = 96 \times h$$

$$h = 176/96$$

$$= 11/6$$

$$= 1.833 \text{ m}$$

$$\approx 1.83 \text{ m}$$

d) Curved surface area of well:

$$= 2\pi rh$$

$$= 2 \times (22/7) \times 2 \times 14$$

$$= 2 \times (22/7) \times 28$$

$$= 2 \times 88$$

$$= 176 \text{ m}^2$$

$$\text{Cost of lining} = 176 \times 25 = ₹4400$$

Answers: a) Soil volume =  $176 \text{ m}^3$  b) Field area =  $96 \text{ m}^2$  c) Rise =  $11/6 \text{ m} \approx 1.83 \text{ m}$  d)  
Lining cost = ₹4400

