

HOTS Questions on Chapter 14 Class 10 Probability

Case Based HOTS Questions on Probability

Question 18: A school organized a quiz competition. 40 students participated 15 from Class 8, 10 from Class 9, and 15 from Class 10. One student is selected as the winner.

a) Find $P(\text{winner is from Class 10})$. b) Find $P(\text{winner is NOT from Class 9})$. c) Find $P(\text{winner is from Class 8 or Class 9})$. d) If the winner must be from Class 9 or 10, what is the probability? e) The school says "a junior student won." If juniors are Classes 8 and 9, find $P(\text{the junior is from Class 9 given the winner is a junior})$.

Answer Explanation:

Total = 40

Class 8 = 15, Class 9 = 10, Class 10 = 15

a) $P(\text{Class 10}) = 15/40 = 3/8$

b) $P(\text{not Class 9}) = (40-10)/40 = 30/40 = 3/4$

c) $P(\text{Class 8 or 9}) = (15+10)/40 = 25/40 = 5/8$

d) $P(\text{Class 9 or 10}) = (10+15)/40 = 25/40 = 5/8$

e) Given: winner is a junior (Class 8 or 9 = 25 students)

$P(\text{Class 9} \mid \text{junior}) = 10/25 = 2/5$

(Out of 25 juniors, 10 are from Class 9)

Answers: a) $3/8$, b) $3/4$, c) $5/8$, d) $5/8$, e) $2/5$

Question 19:

A medical test for a disease has the following record from 10,000 patients:

	Has Disease	No Disease	Total
Test Positive	90	100	190
Test Negative	10	9800	9810
Total	100	9900	10000

Find: a) $P(\text{patient has disease})$ b) $P(\text{test is positive})$ c) $P(\text{test positive AND has disease})$
 d) $P(\text{patient is healthy given test is negative})$

Answer Explanation:

Total = 10,000

a) $P(\text{has disease}) = 100/10000 = 1/100$

b) $P(\text{test positive}) = 190/10000 = 19/1000$

c) $P(\text{positive AND disease}) = 90/10000 = 9/1000$

d) $P(\text{healthy} | \text{test negative})$:

Test negative total = 9810

Healthy AND test negative = 9800

$P = 9800/9810 = 980/981 \approx 0.9990$

So if test is negative, there is a 99.9% chance the patient is actually healthy!

This shows the test is quite reliable for negative results.

Answers: a) $1/100$, b) $19/1000$, c) $9/1000$, d) $980/981 \approx 99.9\%$

Question 20: At a fair, three games are played:

Game 1: Spinning a wheel divided into 8 equal sections (numbered 1 - 8). Win if number is prime.

Game 2: Drawing a card from 20 cards numbered 1 - 20. Win if card is a multiple of 4.

Game 3: Rolling a die. Win if number is greater than 4.

a) Find the probability of winning each game. b) Which game gives the best chance of winning? c) If you play all three independently, find $P(\text{winning all three})$. d) Find $P(\text{winning at least one game})$.

Answer:

Game 1: Wheel 1 - 8, primes = $\{2,3,5,7\} = 4$ numbers

$$P(\text{win Game 1}) = 4/8 = 1/2$$

Game 2: Cards 1-20, multiples of 4 = $\{4,8,12,16,20\} = 5$ cards

$$P(\text{win Game 2}) = 5/20 = 1/4$$

Game 3: Die, numbers $> 4 = \{5,6\} = 2$ numbers

$$P(\text{win Game 3}) = 2/6 = 1/3$$

b) Best chance = Game 1 (probability $1/2$ is highest)

$$\text{c) } P(\text{win all three}) = (1/2) \times (1/4) \times (1/3)$$

$$= 1/24$$

d) $P(\text{win at least one})$:

$$= 1 - P(\text{lose all three})$$

$$= 1 - P(\text{lose G1}) \times P(\text{lose G2}) \times P(\text{lose G3})$$

$$= 1 - (1/2) \times (3/4) \times (2/3)$$

$$= 1 - 6/24$$

$$= 1 - 1/4$$

$$= 3/4$$

Answers: a) $1/2$, $1/4$, $1/3$ | b) Game 1 | c) $1/24$ | d) $3/4$

Mixed HOTS Practice Questions on Probability

Question 21: A number is selected from the first 50 natural numbers. Find the probability that it is: a) Divisible by 4 or 6 b) Divisible by 4 but not 6 c) Divisible by neither 4 nor 6

Answer:

$$n(S) = 50$$

Multiples of 4 (1 - 50): 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48 = 12

Multiples of 6 (1 - 50): 6, 12, 18, 24, 30, 36, 42, 48 = 8

Multiples of both (LCM = 12): 12, 24, 36, 48 = 4

a) Divisible by 4 OR 6:

$$= 12 + 8 - 4 = 16$$

$$P = 16/50 = 8/25$$

b) Divisible by 4 but NOT 6:

= Multiples of 4 - Multiples of both

$$= 12 - 4 = 8$$

$$P = 8/50 = 4/25$$

c) Neither 4 nor 6:

$$= 50 - 16 = 34$$

$$P = 34/50 = 17/25$$

Answers: a) $8/25$, b) $4/25$, c) $17/25$

Question 22: A bag contains 5 red, 8 blue, and 7 green balls. Two balls are drawn at random (without replacement). Find: a) P(both red) b) P(one red, one blue) c) P(at least one green)

Answer:

Total = 20 balls

a) P(both red):

$$= (5/20) \times (4/19)$$

$$= 20/380$$

$$= 1/19$$

b) P(one red, one blue):

$$= P(\text{red first, blue second}) + P(\text{blue first, red second})$$

$$= (5/20)(8/19) + (8/20)(5/19)$$

$$= 40/380 + 40/380$$

$$= 80/380$$

$$= 4/19$$

c) P(at least one green):

$$= 1 - P(\text{no green})$$

$$= 1 - P(\text{both non-green})$$

Non - green = 5+8 = 13 balls

$$= 1 - (13/20)(12/19)$$

$$= 1 - 156/380$$

$$= 1 - 39/95$$

$$= 56/95$$

Answers: a) 1/19, b) 4/19, c) 56/95

Question 23: In a group of 100 people: 60 speak Hindi, 40 speak English, and 20 speak both. A person is chosen at random.

a) P(speaks Hindi only) b) P(speaks English only) c) P(speaks exactly one language) d) P(speaks at least one language) e) P(speaks neither)

Answer:

Total = 100

Hindi only = $60 - 20 = 40$

English only = $40 - 20 = 20$

Both = 20

Neither = $100 - (40+20+20) = 100 - 80 = 20$

a) $P(\text{Hindi only}) = 40/100 = 2/5$

b) $P(\text{English only}) = 20/100 = 1/5$

c) $P(\text{exactly one language})$:

$= (40 + 20)/100 = 60/100 = 3/5$

d) $P(\text{at least one language})$:

$= (40 + 20 + 20)/100 = 80/100 = 4/5$

OR = $1 - P(\text{neither}) = 1 - 20/100 = 4/5$

e) $P(\text{neither}) = 20/100 = 1/5$

Answers: a) $2/5$, b) $1/5$, c) $3/5$, d) $4/5$, e) $1/5$

Question 24: A piggy bank contains: 50 paise coins (100), ₹1 coins (50), ₹2 coins (20), ₹5 coins (30). One coin is picked randomly.

a) $P(\text{coin is ₹1 or ₹2})$ b) $P(\text{coin value} \geq ₹2)$ c) $P(\text{coin value} < ₹1)$ d) What is the expected value of the randomly picked coin?

Answer:

Total coins = $100 + 50 + 20 + 30 = 200$

a) $P(₹1 \text{ or } ₹2) = (50+20)/200 = 70/200 = 7/20$

$$b) P(\text{value} \geq ₹2) = (20+30)/200 = 50/200 = 1/4$$

$$c) P(\text{value} < ₹1) = P(50 \text{ paise}) = 100/200 = 1/2$$

d) Expected value:

$$= (100/200) \times 0.50 + (50/200) \times 1 + (20/200) \times 2 + (30/200) \times 5$$

$$= 0.5 \times 0.50 + 0.25 \times 1 + 0.1 \times 2 + 0.15 \times 5$$

$$= 0.25 + 0.25 + 0.20 + 0.75$$

$$= ₹1.45$$

Answers: a) $7/20$, b) $1/4$, c) $1/2$, d) ₹1.45

Question 25: Cards are numbered 1 to 100. One card is drawn randomly.

a) P(number is a perfect cube) b) P(number is both perfect square and perfect cube) c) P(number is a perfect square or perfect cube) d) Find the probability that the number is a 2-digit number greater than 50 and divisible by 7.

Answer Explanation:

$$n(S) = 100$$

a) Perfect cubes from 1-100:

$$1, 8, 27, 64 = 4 \text{ numbers}$$

$$P = 4/100 = 1/25$$

b) Both perfect square and perfect cube

= Perfect sixth powers:

$$1, 64 = 2 \text{ numbers } (1 = 1^6, 64 = 2^6)$$

$$P = 2/100 = 1/50$$

c) Perfect squares (1 - 100):

$$1, 4, 9, 16, 25, 36, 49, 64, 81, 100 = 10$$

Perfect cubes = 4

Both = 2

By inclusion-exclusion:

$$= 10 + 4 - 2 = 12$$

$$P = 12/100 = 3/25$$

d) 2 - digit numbers > 50 divisible by 7:

Range: 51 to 99

Multiples of 7: 56, 63, 70, 77, 84, 91, 98 = 7 numbers

$$P = 7/100$$

Answers: a) 1/25, b) 1/50, c) 3/25, d) 7/100

