HOTS Questions on Chapter 2 'Polynomials' for Class 10

Question 1: The squared difference of zeroes of the polynomial $x^2 + px + 45$ equals 144. Find all possible values of p.

Solution:

Let zeroes be α and β.

From $x^2 + px + 45$: $\alpha + \beta = -p \quad \alpha\beta = 45$

Given: $(\alpha - \beta)^2 = 144$

$(\alpha + \beta)^2 - 4\alpha\beta = 144$

$(-p)^2 - 4(45) = 144$

$p^2 - 180 = 144$

$p^2 = 324$

$p = \pm 18$

$\therefore$ p = 18 or p = −18

Question 2: α and β are zeroes of $p(x) = x^2 + 8x + 6$. Form a quadratic polynomial whose zeroes are α/β and β/α.

Solution:

From $x^2 + 8x + 6$: $\alpha + \beta = -8 \quad \alpha\beta = 6$

New sum: $\alpha/\beta + \beta/\alpha = (\alpha^2 + \beta^2)/(\alpha\beta) = [(\alpha+\beta)^2 - 2\alpha\beta]/(\alpha\beta)$

$= [64 - 12]/6 = 52/6 = 26/3$

New product: $(\alpha/\beta)(\beta/\alpha) = 1$

New polynomial: $x^2 - (26/3)x + 1 = (1/3)(3x^2 - 26x + 3)$

Required polynomial: $3x^2 - 26x + 3$

Question 3: If α and β are the zeroes of the polynomial $2x^2 - 4x + 5$, find the values of: (a) $\alpha^2 + \beta^2$, (b) $(\alpha - \beta)^2$, and (c) $\alpha^3 + \beta^3$.

Solution:

From the polynomial $2x^2 - 4x + 5$:

$\alpha + \beta = 4/2 = 2 \quad \alpha\beta = 5/2 = 2.5$

(a) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (2)^2 - 2(5/2) = 4 - 5 = -1$

(b) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$= 4 - 4(5/2) = 4 - 10 = -6$

(c) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$
$= (2)^3 - 3(5/2)(2) = 8 - 15 = -7$

Question 4: If one zero of the quadratic polynomial $(k^2 + k)x^2 + 68x + 6k$ is the reciprocal of the other, find the value of k.
Solution: If one zero is α, the other is 1/α.
Product $= \alpha \times (1/\alpha) = 1$
From the polynomial: Product of zeroes $= 6k/(k^2 + k)$
$6k/(k^2 + k) = 1$
$\Rightarrow 6k = k^2 + k$
$\Rightarrow k^2 - 5k = 0$
$\Rightarrow k(k - 5) = 0$
k = 0 or k = 5
Check k = 0: Leading coefficient = 0 → not a valid polynomial.
So k = 5.
∴ k = 5

Question 5: Find a cubic polynomial with sum of zeroes = 3/2, sum of product of zeroes taken two at a time = −1, and product of zeroes = −1/4.
Solution:
Using the cubic template: $k[x^3 - (S_1)x^2 + (S_2)x - S_3]$
$S_1 = 3/2$, $S_2 = -1$, $S_3 = -1/4$
Polynomial $= k[x^3 - (3/2)x^2 + (-1)x - (-1/4)]$
$= k[x^3 - (3/2)x^2 - x + 1/4]$
Multiply by 4 (set k = 4): $4x^3 - 6x^2 - 4x + 1$
Cubic polynomial: $4x^3 - 6x^2 - 4x + 1$

Question 6: If the zeroes of the polynomial $f(x) = x^3 - 12x^2 + 39x + a$ are in an arithmetic progression, find the value of a.
Solution:
Let zeroes in AP be (m−d), m, (m+d)
Sum of zeroes: $(m-d) + m + (m+d) = 3m = 12$
$\Rightarrow m = 4$
Sum of pairwise products: $(m-d)m + m(m+d) + (m-d)(m+d) = 39$
$m^2 - md + m^2 + md + m^2 - d^2 = 39$

$\Rightarrow 3m^2 - d^2 = 39$
$\Rightarrow 3(16) - d^2 = 39$
$\Rightarrow d^2 = 48 - 39 = 9$
$\Rightarrow d = \pm 3$
Zeroes are: 1, 4, 7 (or 7, 4, 1 — same set)
Product of zeroes: $1 \times 4 \times 7 = 28 = -a/1$
$\Rightarrow a = -28$
$\therefore a = -28$

Question 7: On dividing polynomial $4x^4 - 5x^3 - 39x^2 - 46x - 2$ by polynomial $g(x)$, the quotient is $x^2 - 3x - 5$ and the remainder is $-5x + 8$. Find $g(x)$.
Solution:
From Division Algorithm: $p(x) = g(x) \cdot q(x) + r(x)$
So: $g(x) = [p(x) - r(x)] \div q(x)$
$p(x) - r(x) = 4x^4 - 5x^3 - 39x^2 - 46x - 2 - (-5x + 8)$
$= 4x^4 - 5x^3 - 39x^2 - 41x - 10$
Divide by $q(x) = x^2 - 3x - 5$:
$(4x^4 - 5x^3 - 39x^2 - 41x - 10) \div (x^2 - 3x - 5) = 4x^2 + 7x + 2$
$g(x) = 4x^2 + 7x + 2$

Question 8: Obtain all zeroes of $p(x) = 3x^4 - 15x^3 + 17x^2 + 5x - 6$, given that two of its zeroes are $-1/\sqrt{3}$ and $1/\sqrt{3}$.
Solution:
Step 1: Known zeroes are $\pm 1/\sqrt{3}$. Form their factor:
$(x - 1/\sqrt{3})(x + 1/\sqrt{3}) = x^2 - 1/3$
Multiply by 3: $3x^2 - 1$
Step 2: Divide $p(x)$ by $(3x^2 - 1)$:
$(3x^4 - 15x^3 + 17x^2 + 5x - 6) \div (3x^2 - 1) = x^2 - 5x + 6$
Step 3: Factorise $x^2 - 5x + 6 = (x - 2)(x - 3)$
All four zeroes: $-1/\sqrt{3}$, $1/\sqrt{3}$, 2, 3

Question 9: If 1 and −1 are both zeroes of polynomial $Lx^4 + Mx^3 + Nx^2 + Rx + P$, show that $L + N + P = M + R = 0$.
Solution:
Since 1 is a zero: $p(1) = L + M + N + R + P = 0$ ...(i)

Since -1 is a zero: $p(-1) = L - M + N - R + P = 0$ ...(ii)
Adding (i) and (ii): $2L + 2N + 2P = 0 \Rightarrow L + N + P = 0$
Subtracting (ii) from (i): $2M + 2R = 0 \Rightarrow M + R = 0$
Therefore: $L + N + P = M + R = 0$

Question 10: Is the polynomial $y^4 + 4y^2 + 5$ guaranteed to have zeroes? Justify your answer using the discriminant or graphical reasoning.

Solution:

Substitute $t = y^2$: $p = t^2 + 4t + 5$.
For real zeroes of t: discriminant $= 16 - 20 = -4 < 0$. So t has no real values.
Since the discriminant is negative, there are no real values of t satisfying the equation. Therefore, there is no real y such that $y^4 + 4y^2 + 5 = 0$. Hence, the polynomial has no real zeroes.
Graph reasoning: $y^4 \geq 0$, $4y^2 \geq 0$, and adding 5 means the minimum value is 5 (at $y = 0$). The graph never touches the x-axis.