HOTS Questions on Chapter 3 'Pair of Linear Equations in Two Variables' for Class 10

Question 1: For what value of k will the pair of equations $x - ky + 4 = 0$ and $2x - 4y - 8 = 0$ be inconsistent?

Solution:

For the system to be inconsistent (no solution):

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Here: $a_1/a_2 = 1/2$, $b_1/b_2 = -k/-4 = k/4$

Setting $a_1/a_2 = b_1/b_2$:

$1/2 = k/4 \Rightarrow k = 2$

Answer: $k = 2$

Question 2: For what value of m will the system $mx + 3y = m - 3$ and $12x + my = m$ have no solution?

Solution:

For no solution: $a_1/a_2 = b_1/b_2 \neq c_1/c_2$

$m/12 = 3/m \Rightarrow m^2 = 36 \Rightarrow m = \pm 6$

Check $m = 6$: $c_1/c_2 = (6-3)/6 = 3/6 = 1/2$; $b_1/b_2 = 3/6 = 1/2$

$\Rightarrow$ Equal, so this gives infinite solutions, not no solution.

Check $m = -6$: $c_1/c_2 = (-6-3)/(-6) = -9/-6 = 3/2$; $b_1/b_2 = 3/-6 = -1/2$

$\Rightarrow 3/2 \neq -1/2$

Answer: $m = -6$

Question 3: The sum of two numbers is 8. If their sum is four times their difference, find the numbers.

Solution:

Let the two numbers be x and y ($x > y$).

$x + y = 8 \ldots (1)$

$x + y = 4(x - y)$

$8 = 4x - 4y$

$4x - 4y = 8 \Rightarrow x - y = 2 \ldots (2)$

Adding (1) and (2): $2x = 10 \Rightarrow x = 5$

From (1): $y = 8 - 5 = 3$

The two numbers are 5 and 3.

Question 4: In a deer park, the number of heads and legs of deer and human visitors were counted. There were 39 heads and 132 legs. Find the number of deer and the number of human visitors.

Solution:

Let number of deer = x, number of humans = y.

$x + y = 39$ … (1) [Total heads]

$4x + 2y = 132$ … (2) [Total legs]

From (1): $y = 39 - x$

Substituting in (2): $4x + 2(39 - x) = 132$

$4x + 78 - 2x = 132$

$2x = 54 \Rightarrow x = 27$

$y = 39 - 27 = 12$

Number of deer = 27; Number of human visitors = 12.

Question 5: Solve the following pair of equations:

$103x + 51y = 617$

$97x + 49y = 583$

Solution:

Adding both equations:

$200x + 100y = 1200$

$\Rightarrow 2x + y = 12$ … (3)

Subtracting the second from the first:

$6x + 2y = 34$

$\Rightarrow 3x + y = 17$ … (4)

Subtracting (3) from (4):

$x = 5$

Substituting in (3): $2(5) + y = 12 \Rightarrow y = 2$

Answer: $x = 5, y = 2$

Question 6: 8 men and 12 women can finish a piece of work in 10 days, while 6 men and 8 women can finish it in 14 days. Find the time taken by one man alone and one woman alone to finish the work.

Solution:

Let one man alone take 'a' days and one woman alone take 'b' days.
Work done by one man in 1 day = 1/a
Work done by one woman in 1 day = 1/b
Equation 1: 8/a + 12/b = 1/10 … (1)
Equation 2: 6/a + 8/b = 1/14 … (2)
Let 1/a = p and 1/b = q:
8p + 12q = 1/10
⇒ 80p + 120q = 1 … (1')
6p + 8q = 1/14
⇒ 84p + 112q = 1 … (2')
Multiply (1') by 84 and (2') by 80:
6720p + 10080q = 84
6720p + 8960q = 80
Subtracting: 1120q = 4
⇒ q = 1/280
Substituting: 80p + 120(1/280) = 1
80p = 1 − 3/7 = 4/7
p = 1/140
One man alone can finish the work in 140 days; one woman alone in 280 days.

Question 7: Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." Represent this situation algebraically and find their current ages.
Solution:
Let Aftab's present age = x and daughter's present age = y.
Seven years ago:
x − 7 = 7(y − 7)
x − 7 = 7y − 49
x − 7y = −42 … (1)
Three years from now:
x + 3 = 3(y + 3)
x + 3 = 3y + 9
x − 3y = 6 … (2)
Subtracting (1) from (2):
4y = 48

$\Rightarrow y = 12$
From (2): $x = 6 + 3(12) = 6 + 36 = 42$
Aftab's present age = 42 years; Daughter's present age = 12 years.

Question 8: Cars are parked in a parking lot in equal rows. If 3 more cars are added per row, there would be 1 row fewer. If 3 fewer cars are placed in a row, there would be 2 more rows. Find the total number of cars.
Solution:
Let the number of cars per row = x and number of rows = y.
Total cars = xy
Condition 1: $(x + 3)(y - 1) = xy$
$xy - x + 3y - 3 = xy$
$3y - x = 3 \ldots (1)$

Condition 2: $(x - 3)(y + 2) = xy$
$xy + 2x - 3y - 6 = xy$
$2x - 3y = 6 \ldots (2)$
Adding (1) and (2):
$x = 9$
From (1): $3y - 9 = 3 \Rightarrow 3y = 12 \Rightarrow y = 4$
Total cars = $xy = 9 \times 4 = 36$ cars