HOTS Questions on Chapter 3 'The World of Numbers' for Class 9

Question 1: The set of natural numbers is closed under subtraction. True or False? Give two counterexamples.

Solution:

Answer: FALSE

A set is closed under an operation if performing that operation on any two members always produces a member of the same set.

Counter-example 1: $3 - 5 = -2$. Here, $3 \in \mathbb{N}$ and $5 \in \mathbb{N}$, but $-2 \notin \mathbb{N}$.

Counter-example 2: $1 - 7 = -6$. Both 1 and 7 are natural numbers, but -6 is a negative integer, not a natural number.

Question 2: The temperature in Ladakh at noon is 4°C. By midnight, it drops by 15°C. Write the midnight temperature as an integer and justify using Brahmagupta's debt-fortune model.

Solution:

Answer: Midnight temperature = $4 + (-15) = 4 - 15 = -11$°C

In Brahmagupta's framework, a fortune of 4 (4°C above zero) combined with a debt of 15 (a drop of 15°) gives a net debt of 11 or 11 degrees below zero.

Question 3: Explain why the average of two rational numbers a and b is always a rational number that lies strictly between them.

Solution:

Let a and b be rational numbers with $a < b$.

Average = $(a + b) / 2$

Step 1: Show it is rational. Since a and b are rational, their sum a+b is rational (closed under addition).

Dividing by 2 gives $(a+b)/2$, which is rational (closed under division by non-zero rational).

Step 2: Show it lies between a and b. We need: $a < (a + b)/2 < b$

Left inequality: multiply both sides by 2 $\Rightarrow 2a < a + b \Rightarrow a < b$ (true by assumption)

Right inequality: multiply both sides by 2 $\Rightarrow a + b < 2b \Rightarrow a < b$ (true by assumption)

Question 4: Let a and b be two non-zero rational numbers such that $a/b + 1 = 0$. Without assigning any numerical values, determine whether the product ab is positive or negative. Justify.

Solution: Given: $a/b + 1 = 0 \Rightarrow a/b = -1 \Rightarrow a = -b$

Therefore: $ab = (-b) \times b = -b^2$

Since $b \neq 0$, b^2 is always positive (a non-zero number squared is positive). So $-b^2$ is always negative.

Hence, $ab < 0$ for all valid non-zero rational values of a and b satisfying the given condition.

Question 5: Prove that $\sqrt{5}$ is irrational. Will the same method of proof worl for $\sqrt{4}$? Why or why not?

Solution:

Proof that $\sqrt{5}$ is irrational:

Assume $\sqrt{5} = p/q$ (co-prime, $q \neq 0$).

$\Rightarrow 5q^2 = p^2$

$\Rightarrow p^2$ is divisible by 5 $\Rightarrow$ p is divisible by 5.

Let $p = 5k$.

$\Rightarrow 5q^2 = 25k^2$

$\Rightarrow q^2 = 5k^2$

$\Rightarrow$ q is divisible by 5.

Both p, q are divisible by 5

CONTRADICTION to the assumption that p and q are co-primes.

Hence, $\sqrt{5}$ is irrational.

Does it work for $\sqrt{4}$?

Assume $\sqrt{4} = p/q$ (co-prime, $q \neq 0$).

$\Rightarrow 4q^2 = p^2$

Now p^2 is divisible by 4.

But p could be even with $p = 2k$: $\Rightarrow 4q^2 = 4k^2$

$\Rightarrow q^2 = k^2 \Rightarrow q = k$

This means $p/q = 2k/k = 2$. No contradiction arises

The proof by contradiction for $\sqrt{n}$ works when n is NOT a perfect square. For perfect squares like 4, 9, 16, the square root is rational, so the proof correctly 'fails'.

Question 6: Classify each of the following as rational or irrational:
(i) √81 (ii) √12 iii) 0.333… (iv) 0.12345 12345 12345…
(v) 1.010010001000… (vi) 23.56018561…
Solution:
(i) $\sqrt{81} = 9$ → RATIONAL (perfect square)
(ii) $\sqrt{12} = \sqrt{(4\times3)} = 2\sqrt{3}$ → IRRATIONAL (√3 is irrational; multiplying by rational 2 keeps it irrational)
(iii) $0.333... = 0.\overline{3} = 1/3$ → RATIONAL (repeating decimal)
(iv) 0.12345 12345 12345... The block '12345' repeats → RATIONAL
p/q form: let x = 0.\overline{12345}; then 100000x = 12345.\overline{12345}, 99999x = 12345
$x = 12345/99999 = 823/6667$
(v) 1.010010001000... Pattern: 1 zero, then 2 zeros, then 3 zeros : increasing zeros, NO fixed repeating block → IRRATIONAL
(vi) 23.56018561... (non-terminating, no visible repeating block stated) → IRRATIONAL

Question 7: A rational number in its lowest form has denominator $2^3 \times 5$. How many decimal places will its decimal expansion have? Explain your reasoning.
Solution:
The denominator is $2^3 \times 5^1$. Since it has only factors of 2 and 5, the decimal terminates.
To convert to a power of 10, we need the denominator = 10^n = 2^n × 5^n.
Currently: $2^3 \times 5^1$
Need to multiply by $5^2 = 25$ (to balance the powers): $2^3 \times 5^1 \times 5^2 = 2^3 \times 5^3 = 10^3 = 1000$
So the denominator becomes 1000 → 3 decimal places.
Answer: 3 decimal places.
General Rule: If the denominator in lowest form is 2^m × 5^n, the decimal terminates in max(m, n) decimal places.

Question 8: If $2a = 3b = 6c$, show that $c = ab/(a + b)$
Solution:
Let $2a = 3b = 6c = k$ (some constant).

Then: $2 = k^{(1/a)}$, $3 = k^{(1/b)}$, $6 = k^{(1/c)}$
Since $6 = 2 \times 3$: $k^{(1/c)} = k^{(1/a)} \times k^{(1/b)} = k^{(1/a + 1/b)}$
Therefore: $1/c = 1/a + 1/b$ $1/c = (a + b) / (ab)$
$\Rightarrow c = ab/(a + b)$

Question 9: Three rational numbers x,y,z satisfy $x + y + z = 0$ and $xy + yz + zx = 0$. Show that $x = y = z$
Solution:
Given: $x + y + z = 0 \rightarrow z = -(x + y)$ and $xy + yz + zx = 0$
Substituting $z = -(x+y)$: $xy + y(-x-y) + x(-x-y) = 0$
$\Rightarrow xy - xy - y^2 - x^2 - xy = 0$
$\Rightarrow -x^2 - xy - y^2 = 0$ $x^2 + xy + y^2 = 0$
Multiply by 4: $4x^2 + 4xy + 4y^2 = 0$
$(4x^2 + 4xy + y^2) + 3y^2 = 0$
$\Rightarrow (2x + y)^2 + 3y^2 = 0$
Since $(2x+y)^2 \geq 0$ and $3y^2 \geq 0$, their sum $= 0$ requires $3y^2 = 0$
$\Rightarrow y = 0$
$(2x + y)^2 = 0 \Rightarrow 2x + 0 = 0 \Rightarrow x = 0$
$\Rightarrow z = -(0+0) = 0$
Therefore, $x = y = z = 0$.