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HOTS Questions on Chapter 4 'Exploring Algebraic Identities' for Class 9

Quick Recap: All Identities from Chapter 4

Identity	Expanded Form
$(x+y)^2$	$x^2 + 2xy + y^2$
$(x-y)^2$	$x^2 - 2xy + y^2$
$(x+y+z)^2$	$x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
$(x+y)(x-y)$	$x^2 - y^2$
$(x+a)(x+b)$	$x^2 + (a+b)x + ab$
$(ax+b)(cx+d)$	$acx^2 + (ad+bc)x + bd$
$(x+y)^3$	$x^3 + 3x^2y + 3xy^2 + y^3$
$(x-y)^3$	$x^3 - 3x^2y + 3xy^2 - y^3$
x^3+y^3	$(x+y)(x^2-xy+y^2)$
x^3-y^3	$(x-y)(x^2+xy+y^2)$
$x^3+y^3+z^3-3xyz$	$(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

Solved HOTS Questions on Algebraic Identities

Question 1: If $x + y = 12$ and $xy = 32$, find the value of $x^2 + y^2$.
Solution:
We use the identity:
$(x + y)^2 = x^2 + 2xy + y^2$
So: $x^2 + y^2 = (x + y)^2 - 2xy$

Substituting:
$x^2 + y^2 = (12)^2 - 2(32) = 144 - 64 = 80$

Question 2: Without multiplying directly, evaluate:
(i) 997^2 (ii) 1003×997
Solution:
(i) Write $997 = (1000 - 3)$
$(1000 - 3)^2 = 1000^2 - 2(1000)(3) + 3^2$
$= 1{,}000{,}000 - 6000 + 9 = 994{,}009$
(ii) $1003 \times 997 = (1000 + 3)(1000 - 3)$
Using $(a + b)(a - b) = a^2 - b^2$:
$= 1000^2 - 3^2 = 1{,}000{,}000 - 9 = 999{,}991$

Question 3: For any three consecutive square numbers, prove that adding the smallest and largest, then subtracting twice the middle square, always gives 2.
Solution:
Let the three consecutive integers be $(n - 1)$, n, and $(n + 1)$.
Their squares: $(n - 1)^2$, n^2, $(n + 1)^2$
Sum of smallest and largest:
$(n - 1)^2 + (n + 1)^2$
$= (n^2 - 2n + 1) + (n^2 + 2n + 1)$
$= 2n^2 + 2$
Subtracting twice the middle square:
$2n^2 + 2 - 2n^2 = 2$
Since n can be any integer, this is always 2.

Question 4: Find the value of $x^3 - y^3$ if $x - y = 4$ and $xy = 5$.
Solution: Use $x^3 - y^3 = (x - y)^3 + 3xy(x - y)$
$= (4)^3 + 3(5)(4)$
$= 64 + 60$
$= 124$

Question 5: If $a + b + c = 5$, $ab + bc + ca = 10$, prove that $a^3 + b^3 + c^3 - 3abc = -25$.
Solution:

We use:
$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
Step 1: Find $a^2 + b^2 + c^2$
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
$25 = a^2 + b^2 + c^2 + 20$
$a^2 + b^2 + c^2 = 5$
Step 2: Find $a^2 + b^2 + c^2 - ab - bc - ca$
$= 5 - 10 = -5$
Step 3: Substitute
$a^3 + b^3 + c^3 - 3abc = (5)(-5) = -25$

Question 6: Check whether $n^3 - n$ is always divisible by 6 for all natural numbers n.
Solution:
Factorise: $n^3 - n = n(n^2 - 1) = n(n - 1)(n + 1)$
This is the product of three consecutive integers: $(n - 1)$, n, $(n + 1)$.
Among any three consecutive integers:
At least one is divisible by 2
Exactly one is divisible by 3
Therefore, their product is divisible by both 2 and 3, hence divisible by 6.

Question 7: Factorise $27u^3 - 1/125 - 27u^2/5 + 9u/25$
Solution:
Rewrite as:
$(3u)^3 - (1/5)^3 - 3(3u)^2(1/5) + 3(3u)(1/5)^2$
$= (3u - 1/5)^3$
Using $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
Answer: $(3u - 1/5)^3$

Question 8: Simplify $(x^2 - 7x + 12) / (5x^2 + 5x - 100)$
Solution:
Numerator: $x^2 - 7x + 12$
Find a, b such that $a + b = -7$ and $ab = 12 \rightarrow a = -3, b = -4$
$= (x - 3)(x - 4)$
Denominator: $5x^2 + 5x - 100 = 5(x^2 + x - 20)$

Find a, b such that $a + b = 1$ and $ab = -20 \rightarrow a = 5, b = -4$
$= 5(x + 5)(x - 4)$
Simplify:
$= (x - 3)(x - 4) / [5(x + 5)(x - 4)]$
$= (x - 3) / [5(x + 5)]$ (cancelling $(x - 4)$, valid since denominator $\neq 0$)

Question 9: If $x + y = -4$, find the value of $x^3 + y^3 - 12xy + 64$.
Solution:
Since $x + y = -4$, we have $x + y + 4 = 0$.
Let $a = x, b = y, c = 4$. Then $a + b + c = 0$.
When $a + b + c = 0$, the identity gives:
$a^3 + b^3 + c^3 = 3abc$
So: $x^3 + y^3 + 64 = 3(x)(y)(4) = 12xy$
Therefore: $x^3 + y^3 - 12xy + 64 = 12xy - 12xy = 0$

Question 10: The volume of a cube is given as $p^3 + 6p^2q + 12pq^2 + 8q^3$ cubic units. Find the side length.
Solution:
Compare with the identity $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$:
$p^3 + 6p^2q + 12pq^2 + 8q^3$
$= p^3 + 3(p^2)(2q) + 3(p)(2q)^2 + (2q)^3$
$= (p + 2q)^3$
So the side of the cube is $(p + 2q)$ units