HOTS Questions on Chapter 5 Class 10 Arithmetic Progressions

HOTS Questions Based on Real-Life Applications of Arithmetic Progressions

Financial Planning Problems

Question 1: A person deposits ₹500 in the first month, ₹600 in the second month, ₹700 in the third month, and so on, into a savings account. How much will they have deposited in total after 24 months? In which month will the monthly deposit reach ₹2,000?

Question 2: A person borrows ₹12,000 and agrees to pay back the loan in monthly instalments. They pay ₹100 in the first month, and each subsequent month they pay ₹50 more than the previous month. In how many months will the entire loan be repaid?

Seating Arrangement and Pattern Questions

Question 3: An auditorium has 20 rows of seats. The first row has 12 seats and each subsequent row has 3 more seats than the row in front of it. What is the total number of seats in the auditorium? If tickets cost ₹150 per seat and all tickets are sold, what is the total revenue?

Question 4: Bricks are laid in a pattern where the bottom row has 30 bricks, and each row above has 2 fewer bricks than the row below. If there are 15 rows, how many bricks are in the top row? What is the total number of bricks used?

Growth and Trend Analysis Questions

Question 5: A plant grows 3 cm in the first week, 4.5 cm in the second week, 6 cm in the third week, and continues growing in this pattern. In which week will the plant have grown more than 20 cm in a single week? What is the total growth of the plant over 10 weeks?

Answer Explanations

Answer 1: Monthly deposits form an AP: 500, 600, 700, ... a = 500, d = 100.

Total after 24 months = S_{24} = 24/2 × [2×500 + 23×100] = 12 × [1000 + 2300] = 12 × 3300 = ₹39,600

Month when deposit reaches ₹2,000: T_n = 500 + (n−1)×100 = 2000 (n−1)×100 = 1500 n−1 = 15 n = 16th month

Answer 2: Monthly payments form an AP: 100, 150, 200, ... a = 100, d = 50.

S_n = n/2 × [2×100 + (n−1)×50] = 12000

n × [200 + 50n − 50] = 24000 n × [50n + 150] = 24000 $50n^2 + 150n = 24000$ $n^2 + 3n = 480$ $n^2 + 3n - 480 = 0$

Using the quadratic formula: n = [−3 ± √(9 + 1920)] / 2 = [−3 ± √1929] / 2

√1929 ≈ 43.9, so n ≈ (−3 + 43.9)/2 ≈ 20.45.

Since n must be a whole number, checking n = 20: S_{20} = 20/2 × [200 + 19×50] = 10 × [200 + 950] = 10 × 1150 = 11,500 (not enough).

n = 21: S_{21} = 21/2 × [200 + 20×50] = 21/2 × 1200 = 12,600 > 12,000.

So after 20 months, ₹11,500 is paid. The remaining ₹500 is paid in month 21.

The loan is fully repaid in 21 months.

Answer 3: Seats per row: 12, 15, 18, ... (AP with a = 12, d = 3, n = 20)

Top row (row 20) = T_{20} = 12 + 19 × 3 = 12 + 57 = 69 seats.

Total seats = S_{20} = 20/2 × (12 + 69) = 10 × 81 = 810 seats

Total revenue = 810 × 150 = ₹1,21,500

Answer 4: Bricks per row from bottom: 30, 28, 26, ... (AP with a = 30, d = −2, n = 15)

Top row = T_{15} = 30 + 14 × (−2) = 30 − 28 = 2 bricks

Total bricks = S_{15} = 15/2 × (30 + 2) = 15/2 × 32 = 15 × 16 = 240 bricks

Answer 5: Weekly growth forms an AP: 3, 4.5, 6, ... (a = 3, d = 1.5)

Week when growth exceeds 20 cm: T_n > 20 3 + (n−1) × 1.5 > 20 (n−1) × 1.5 > 17 n−1 > 11.33 n > 12.33

So from week 13 onwards, the weekly growth exceeds 20 cm. T_{13} = 3 + 12 × 1.5 = 3 + 18 = 21 cm > 20 cm

Total growth over 10 weeks = S_{10} = 10/2 × [2×3 + 9×1.5] = 5 × [6 + 13.5] = 5 × 19.5 = 97.5 cm

HOTS Questions Involving Reverse Thinking

Finding Unknown AP Conditions

Question 1: The sum of first 20 terms of an AP is 400 and the sum of first 40 terms is 1600. Find the sum of terms from the 21st to the 40th term.

Question 2: If the sum of the first n terms of an AP is $5n^2 - 3n$, find the value of n for which the nth term equals 97.

Question 3: Three numbers are in AP. Their sum is 24 and the sum of their squares is 200. Find the three numbers. Are there two possible sets of answers?

Error Detection Questions

Question 4: A student finds the sum of the AP 5, 8, 11, ..., 50 as follows:

"Number of terms n: T_n = 50, so 5 + (n−1)×3 = 50, giving n = 16. Sum = 16/2 × (5 + 50) = 8 × 55 = 440."

Find and explain any error in this solution. What is the correct answer?

Question 5: A student claims that in any AP where the first term is positive and the common difference is negative, the AP will eventually reach zero and then continue with negative terms. Is this always true? Give a specific example to support or counter this claim.

Answer Explanations

Answer 1: S_{20} = 400, S_{40} = 1600.

Sum from 21st to 40th term = $S_{40} - S_{20}$ = 1600 − 400 = 1200.

Notice that the sum of the second group of 20 terms (1200) is 3 times the sum of the first 20 terms (400). This is a known property of APs: in equal blocks of terms, the sums form a new AP with their own constant increase.

Answer 2: $S_n = 5n^2 - 3n$

$T_n = S_n - S_{n-1} = [5n^2 - 3n] - [5(n-1)^2 - 3(n-1)] = 5n^2 - 3n - 5n^2 + 10n - 5 + 3n - 3 = 10n - 8$

Set $T_n = 97$: $10n - 8 = 97$ $10n = 105$ $n = 10.5$

Since n must be a positive integer, there is no term in this AP that equals exactly 97. This means 97 is NOT a term in this AP. (This is itself a valid HOTS answer recognising when a value does not appear in the sequence.)

Answer 3: Let the three terms be a−d, a, and a+d.

Sum: 3a = 24, so a = 8.

Sum of squares: $(8-d)^2 + 8^2 + (8+d)^2 = 200$ $64 - 16d + d^2 + 64 + 64 + 16d + d^2 = 200$ $192 + 2d^2 = 200$ $2d^2 = 8$ $d^2 = 4$ $d = \pm 2$

When d = 2: terms are 6, 8, 10. When d = −2: terms are 10, 8, 6.

These give the same three numbers, just in different orders. So there is effectively one set: {6, 8, 10}, written either in ascending or descending order.

Verification: 6 + 8 + 10 = 24 and 36 + 64 + 100 = 200

Answer 4: The student's method is correct in approach, but let us verify the number of terms calculation.

$T_n = 5 + (n-1) \times 3 = 50$ $(n-1) \times 3 = 45$ $n - 1 = 15$ $n = 16$

Sum = 16/2 × (first + last) = 8 × (5 + 50) = 8 × 55 = 440

Actually, the student's solution is correct. The sum of the AP 5, 8, 11, ..., 50 is 440. This question tests whether students can evaluate an existing solution critically recognising that it is correct requires as much skill as finding an error.

Answer 5: The student's claim is not always true.

The AP will reach zero only if zero is actually a term in the sequence, which requires (a − nd) = 0 for some positive integer n, that is, n = a/d must be a positive integer.

Counter example: Consider the AP with a = 5 and d = −2: sequence is 5, 3, 1, −1, −3, ...

Here 0 is never reached the sequence jumps from 1 to −1, skipping zero entirely. So the AP goes from positive to negative without ever equalling zero.

However, if a = 4 and d = −2: sequence is 4, 2, 0, −2, −4, ... Here zero IS a term.

Conclusion: An AP with positive first term and negative common difference will always eventually become negative, but it will only pass through zero if a/d is a positive integer. Otherwise, it skips zero and goes directly from a positive term to a negative term.

Mixed HOTS Practice Questions on Arithmetic Progressions

Advanced HOTS Questions

Question 1: The first and last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are in the AP? What is the sum of all the terms?

Question 2: The 5th and 15th terms of an AP are 13 and 43 respectively. Find all terms of the AP that are divisible by 6.

Question 3: If the mth term of an AP is 1/n and the nth term is 1/m, find the (mn)th term.

Challenge-Based Problems

Question 4: A contractor hires workers for a project. On the first day, 5 workers are hired. Each day, 3 more workers are added. If the project needs a total of 480 worker-days (total work done by all workers across all days), in how many days will the project be completed?

Question 5: The digits of a three-digit number are in AP. The number itself is 8 times the sum of its digits. Find the number.

Multi-Step Reasoning Questions

Question 6: If the sum of p terms of an AP is the same as the sum of q terms of the same AP, show that the sum of (p+q) terms is zero. What further conclusion can you draw about the (p+q)th term if also p+q ≠ 0?

Answers:

Answer 1: T□ = 350: 17 + (n−1)×9 = 350 → 9(n−1) = 333 → n−1 = 37 → n = 38 terms

Sum = 38/2 × (17 + 350) = 19 × 367 = 6973

Answer 2: a = ?, d = ?

$T_5 = a + 4d = 13$ ... (i) $T_{15} = a + 14d = 43$ ... (ii)

Subtract: $10d = 30$, $d = 3$. From (i): $a = 13 - 12 = 1$.

AP: 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, ...

General term: $T_n = 1 + (n-1)\times3 = 3n - 2$.

For T_n to be divisible by 6: $3n - 2 \equiv 0 \pmod 6$ $3n \equiv 2 \pmod 6$

Testing values: n=2: $T_2=4$ (not div by 6). n=4: $T_4=10$ (no). n=6: $T_6=16$ (no). n=8: $T_8=22$ (no). n=10: $T_{10}=28$ (no). n=12: $T_{12}=34$ (no).

Conclusion: No terms of this AP are divisible by 6. This is itself the HOTS insight — recognising when no solution exists requires higher-order thinking.

Answer 3: $T_m = a + (m-1)d = 1/n$ ... (i) $T_n = a + (n-1)d = 1/m$ ... (ii)

Subtract (ii) from (i): $(m-n)d = 1/n - 1/m = (m-n)/mn$

Since $m \neq n$, divide: $d = 1/mn$

From (i): $a + (m-1)/mn = 1/n$ $a = 1/n - (m-1)/mn = m/mn - (m-1)/mn = 1/mn$

$T(mn) = a + (mn-1)d = 1/mn + (mn-1)/mn = mn/mn = 1$

Answer 4: Workers per day: 5, 8, 11, ... (AP, a = 5, d = 3)

After n days, total worker-days = $S_n = n/2 \times [2\times5 + (n-1)\times3] = n/2 \times [10 + 3n - 3] = n/2 \times [3n + 7]$

Set $S_n = 480$: $n(3n + 7) = 960$ $3n^2 + 7n - 960 = 0$

Discriminant = $49 + 4\times3\times960 = 49 + 11520 = 11569 = \sqrt{11569} \approx 107.6$

$n = (-7 + 107.6)/6 \approx 16.77$

Check n = 16: $S_{16} = 16/2 \times (30+7\times3)$ = wait, let me recalculate: $S_{16} = 16/2 \times [3(16)+7] = 8 \times 55 = 440 < 480$. $S_{17} = 17/2 \times [3(17)+7] = 17/2 \times 58 = 17 \times 29 = 493 > 480$.

The project reaches 480 worker-days during day 17 (the 493rd worker-day is reached by end of day 17, with only 40 additional worker-days needed on day 17, which has 5+16×3=53 workers available).

Answer 5: Let the three digits in AP be a−d, a, a+d.

The three-digit number = 100(a−d) + 10a + (a+d) = 111a − 99d.

Sum of digits = (a−d) + a + (a+d) = 3a.

Condition: number = 8 × sum of digits: 111a − 99d = 8 × 3a = 24a 87a = 99d d = 87a/99 = 29a/33

For d to be an integer and a single digit, a must be divisible by 33. But a is a single digit (1−9), so this has no solution unless a = 33... which is not a digit.

Reconsidering: taking d = 29a/33, for d to be integer, try a = 3: d = 87/33 not integer. Try a = 6: d = 174/33 not integer.

Let me try without subtracting: number = 8 × digit sum.

If digits are in AP as (a−d), a, (a+d): Number = 100(a−d) + 10a + (a+d) = 100a−100d+10a+a+d = 111a−99d. Digit sum = 3a. 111a − 99d = 24a, 87a = 99d, 29a = 33d.

So 29a/33 = d. For integer solution, a must be a multiple of 33 impossible for a digit.

The answer is: no such three-digit number exists under these conditions. This is itself a valid HOTS conclusion proving the non-existence of a solution.