HOTS Questions on Chapter 6 Class 10 Triangles

HOTS Questions Based on Triangle Proofs

Q1: A student writes this proof that triangle ABC is similar to triangle ADB (where D is the foot of the altitude from A to BC in a right triangle with right angle at A). Find the missing step.

"In triangle ABC and triangle ADB: Step 1: angle ADB = angle BAC = 90°. Step 2: ___________ Step 3: By AA, the triangles are similar."

Answer: The missing Step 2 is: angle ABD = angle ABC (common angle at B, shared by both triangles).

With two angles equal (angle ADB = angle BAC = 90°, and angle ABD = angle ABC), the AA criterion is complete.

Q2: A student claims: "Triangle ABC has sides 3, 4, 5 and triangle DEF has sides 6, 8, 9. These are similar because the first two pairs of sides are in ratio 1:2." Is the student correct?

Answer: No. The student is incorrect.

For SSS similarity, ALL three pairs of corresponding sides must be in the same ratio.

Check: 3/6 = 1/2, 4/8 = 1/2, but 5/9 ≠ 1/2.

Since the third pair of sides is not in ratio 1:2, the triangles are not similar. The student made the error of checking only two pairs of sides.

Q3: Given: DE ∥ BC in triangle ABC, with D on AB and E on AC. Prove that AD/AB = AE/AC.

Answer: Since DE ∥ BC, by BPT: AD/DB = AE/EC.

Let AD = m and DB = n, so AB = m + n.

AD/DB = m/n.

AE/EC = m/n (from BPT).

Let AE = mk and EC = nk for some constant k.

AD/AB = m/(m+n).

AE/AC = mk/(mk+nk) = mk/[k(m+n)] = m/(m+n).

Therefore: AD/AB = AE/AC

This result is a powerful extended form of BPT, often used in advanced problems.

Q4: In triangle ABC, the bisector AD of angle A meets BC at D. Prove that AB/AC = BD/DC.

Answer: This is the Angle Bisector Theorem.

Draw CE parallel to DA, meeting BA extended at E.

Since AD ∥ CE (by construction): angle DAB = angle AEC (corresponding angles) and angle DAC = angle ACE (alternate interior angles).

But AD bisects angle A: angle DAB = angle DAC.

Therefore angle AEC = angle ACE, triangle ACE is isosceles, AC = AE.

In triangle BCE: DA ∥ CE (by construction), so by BPT:

BD/DC = BA/AE = BA/AC = AB/AC.

Therefore: AB/AC = BD/DC

Mixed HOTS Practice Questions on Triangles

Q1: In triangle ABC, DE ∥ BC with D on AB and E on AC. If AD:AB = 2:5, find the ratio of the area of triangle ADE to the area of trapezium BCED.

Answer: AD/AB = 2/5, scale factor = 2/5.

Area of triangle ADE / Area of triangle ABC = $(2/5)^2$ = 4/25.

Area of trapezium BCED = Area of ABC − Area of ADE = (25−4)/25 of ABC = 21/25.

Ratio = (4/25) : (21/25) = 4 : 21.

Q2: The perimeters of two similar triangles are 30 cm and 45 cm. If one side of the smaller triangle is 8 cm, find the corresponding side of the larger triangle.

Answer: Ratio of perimeters = 30/45 = 2/3.

Since triangles are similar, ratio of sides = 2/3.

$8/x = 2/3$

$x = 12$ cm.

Q3: Prove that in a right triangle, the square on the hypotenuse equals the sum of squares on the other two sides using the concept of similar triangles.

Answer: In right triangle ABC (right angle at C), draw altitude CD to AB.

Triangle ACD is similar to triangle ABC (AA: angle A common, angle ACD = angle ACB = 90°).

So $AC/AB = AD/AC$, $AC^2 = AB \times AD$ ... (i)

Triangle CBD is similar to triangle ABC (AA: angle B common, angle BCD = angle BCA = 90°).

So $BC/AB = DB/BC$, $BC^2 = AB \times DB$ ... (ii)

Adding (i) and (ii):

$AC^2 + BC^2 = AB \times AD + AB \times DB = AB \times (AD + DB) = AB \times AB = AB^2$

This is the Pythagorean Theorem proved through similar triangles.

Q4: Triangle XYZ has XY = 12 cm, YZ = 16 cm, XZ = 20 cm. A line PQ is drawn parallel to YZ with P on XY and Q on XZ. If XP = 6 cm, find: (a) PQ, (b) area of triangle XPQ as a fraction of area of triangle XYZ.

Answer: $XP/XY = 6/12 = 1/2$.

Since $PQ \parallel YZ$, triangle XPQ is similar to triangle XYZ (AA criterion).

Scale factor = $XP/XY = 1/2$.

(a) $PQ = YZ \times 1/2 = 16 \times 1/2 = 8$ cm.

(b) Area ratio = $(1/2)^2 = 1/4$.

Area of triangle XPQ = (1/4) × Area of triangle XYZ.

Q5: In two similar triangles, the ratio of corresponding altitudes is 3:5. A side of the first triangle is 9 cm. Find the corresponding side of the second triangle and the ratio of their areas.

Answer: Corresponding altitudes of similar triangles are in the same ratio as corresponding sides.

So ratio of sides = 3:5.

Corresponding side of second triangle = $9 \times 5/3 = 15$ cm.

Ratio of areas = $(3/5)^2 = 9 : 25$.