HOTS Questions on Chapter 7: The Mathematics of Maybe: Introduction to Probability for Class 9

Quick Concept Review of Chapter 7

- **Experiment:** Any action with an uncertain outcome. Tossing a coin, rolling a die, drawing a card.
- **Sample Space (S):** The complete set of all possible outcomes of an experiment.
    - One coin: S = {H, T} → n(S) = 2
    - Two coins: S = {HH, HT, TH, TT} → n(S) = 4
    - One die: S = {1, 2, 3, 4, 5, 6} → n(S) = 6
    - Two dice: S = 36 outcomes
    - A standard deck of cards: n(S) = 52
- **Probability Formula:**
  P(E) = Number of favourable outcomes / Total number of outcomes = n(E) / n(S)
- **Key Rules:**
    - $0 \leq P(E) \leq 1$ always
    - P(E) + P(E') = 1, where E' is the complement (the event NOT happening)
    - P(impossible event) = 0
    - P(certain event) = 1
- **Equally Likely Outcomes:** Each outcome has the same chance of occurring.

Solved HOTS Questions on Probability

Question 1: Three unbiased coins are tossed simultaneously. Find the probability of getting:
(a) Exactly two heads
(b) At least one head
(c) At most two tails
(d) Exactly one tail

Solution:
Sample Space: {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}
Total outcomes: n(S) = 8
(a) Exactly two heads: {HHT, HTH, THH}
⇒ n(E) = 3
P(exactly two heads) = 3/8
(b) At least one head (one or more heads): {HHH, HHT, HTH, THH, HTT, THT, TTH}
⇒ n(E) = 7
P(at least one head) = 7/8
Shortcut: P(at least one head) = 1 − P(no heads) = 1 − 1/8 = 7/8
(c) At most two tails (0, 1, or 2 tails): {HHH, HHT, HTH, THH, HTT, THT, TTH}
⇒ n(E) = 7
P(at most two tails) = 7/8
(d) Exactly one tail: {HHT, HTH, THH}
⇒ n(E) = 3
P(exactly one tail) = 3/8

Question 2: Two dice are thrown. Given that the sum of the numbers on the two dice is 7, what is the probability that one of the dice shows a 2?
Solution:
Outcomes where sum = 7: {(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)} ⇒ 6 outcomes
Among these, outcomes where at least one die shows 2: {(2,5),(5,2)} ⇒ 2 outcomes
P(one die shows 2 | sum is 7) = 2/6 = 1/3

Question 3: When two dice are thrown together, the product of the numbers appearing on them lies between 8 and 13 (exclusive). Find the probability.
Solution:
We need products: 9, 10, 11, 12 (strictly between 8 and 13)

- Product = 9: (3,3) → 1 way
- Product = 10: (2,5),(5,2) → 2 ways
- Product = 11: Not possible (11 is prime, and dice only go to 6)
- Product = 12: (2,6),(6,2),(3,4),(4,3) → 4 ways

Total: 1+2+4 = 7 outcomes

P = 7/36

Question 4: All queens, jacks, and aces are removed from a pack of 52 cards. The remaining cards are well-shuffled and one is drawn. Find the probability that the card drawn is:
(a) A king
(b) A face card
(c) A card of clubs
Solution:
Cards removed: 4 queens + 4 jacks + 4 aces = 12 cards
Remaining cards: 52 − 12 = 40
(a) Kings remaining: 4 kings (not removed)
P = 4/40 = 1/10
(b) Face cards remaining: After removing queens and jacks, only kings remain as face cards = 4
P = 4/40 = 1/10
(c) Clubs remaining: 13 clubs originally − 1 queen of clubs − 1 jack of clubs − 1 ace of clubs = 10
P = 10/40 = 1/4

Question 5: From a deck of 52 cards, one card is drawn at random. If it is known that the drawn card is red, what is the probability that it is a king?
Solution:
Red cards in a deck: 26 (13 hearts + 13 diamonds)
Red kings: 2 (king of hearts + king of diamonds)
P(king | card is red) = 2/26 = 1/13

Question 6: A box contains cards numbered 6 to 55. A card is drawn at random. Find the probability that the number on the card is:
(a) A perfect square
(b) Divisible by 5
(c) A prime number less than 20
(d) A multiple of both 3 and 7
Solution:
Cards: 6, 7, 8, ..., 55

⇒ Total = 55 − 6 + 1 = 50 cards
(a) Perfect squares from 6 to 55: 9, 16, 25, 36, 49
⇒ 5 numbers
P = 5/50 = 1/10
(b) Divisible by 5: 10, 15, 20, 25, 30, 35, 40, 45, 50, 55
⇒ 10 numbers
P = 10/50 = 1/5
(c) Prime numbers less than 20, in range 6–55: 7, 11, 13, 17, 19
⇒ 5 numbers
P = 5/50 = 1/10
(d) Multiple of both 3 and 7 = multiple of 21: 21, 42
⇒ 2 numbers
P = 2/50 = 1/25

Question 7: In a survey of 200 students, it was found that 60 like cricket, 80 like football, 40 like both, and the rest like neither. A student is chosen at random. Find the probability that the student:
(a) Likes cricket only
(b) Likes football only
(c) Likes neither
(d) Likes at least one sport
Solution:
Students who like cricket only = 60 − 40 = 20
Students who like football only = 80 − 40 = 40
Students who like both = 40
Students who like neither = 200 − (20 + 40 + 40) = 100
(a) P(cricket only) = 20/200 = 1/10
(b) P(football only) = 40/200 = 1/5
(c) P(neither) = 100/200 = 1/2
(d) P(at least one sport) = 1 − 1/2 = 1/2