HOTS Questions on Class 10 Maths Chapter 13 'Statistics'

Question 1: The mean of the following distribution is 18. Find the value of f.

Class Interval	11–13	13–15	15–17	17–19	19–21	21–23	23–25
Frequency	3	6	9	13	*f*	5	4

Solution:
Class midpoints (x_i): 12, 14, 16, 18, 20, 22, 24
$\Sigma f = 3 + 6 + 9 + 13 + f + 5 + 4 = 40 + f$
$\Sigma f_i x_i = 3(12) + 6(14) + 9(16) + 13(18) + f(20) + 5(22) + 4(24)$
$= 36 + 84 + 144 + 234 + 20f + 110 + 96$
$= 704 + 20f$
Mean $= \Sigma f_i x_i / \Sigma f$
$\Rightarrow 18 = (704 + 20f)/(40 + f)$
$18(40 + f) = 704 + 20f$
$\Rightarrow 720 + 18f = 704 + 20f$
$\Rightarrow 16 = 2f$
$\therefore f = 8$

Question 2: A student computes the average of 10 two-digit positive integers. By mistake, one number's digits are swapped. This makes the mean 2.7 less than the original. Find the difference between the digits of that number.
Solution:
Let the digits of the wrongly-written number be 'a' (tens) and 'b' (units), so the number is 10a+b. When swapped, it becomes 10b+a.
Difference in the number = (10a+b) – (10b+a) = 9(a–b)
This reduces the total sum by 9(a–b), so mean reduces by 9(a–b)/10 = 2.7
9(a–b) = 27 $\Rightarrow$ a–b = 3
$\therefore$ Difference between digits = 3

Question 3: The mean of 11 numbers is 35. The mean of the first 6 numbers is 32 and the mean of the last 6 numbers is 37. Find the 6th number.

Solution:

Sum of all 11 = 35 × 11 = 385

Sum of first 6 = 32 × 6 = 192

Sum of last 6 = 37 × 6 = 222

Sum of first 6 + Sum of last 6 = 192 + 222 = 414, but this counts the 6th number twice.

414 = 385 + 6th number

⇒ 6th number = 414 – 385

∴ 6th number = 29

Question 4: The median of the following frequency distribution is 35. Find the value of x and also identify the modal class.

Class	**0–10**	**10–20**	**20–30**	**30–40**	**40–50**	**50–60**	**60–70**
Frequency	2	3	*x*	6	5	3	2

Solution:

Σf = 2 + 3 + x + 6 + 5 + 3 + 2 = 21 + x.

Since median = 35 lies in 30–40, median class = 30–40.

cf before 30 - 40 = 2 + 3 + x = 5 + x

f = 6; l = 30; h = 10; n/2 = (21 + x)/2

35 = 30 + [((21 + x)/2 – (5 + x)) / 6] × 10

5 = [(21 + x – 10 – 2x)/2 / 6] × 10

⇒ 5 = [(11 – x)/12] × 10

⇒ 6 = (11 – x) × 10/12

⇒ 6×12 = 10(11 – x)

⇒ 72 = 110 – 10x

⇒ 10x = 38

⇒ x = 3.8 ≈ 4 (round to nearest whole number or exact value x = 4)

With x = 4: frequencies are 2,3,4,6,5,3,2
⇒ Maximum frequency = 6 in class 30 - 40.
∴ x = 4, Modal Class = 30 - 40

Question 5: The mean of the following data is 62.8 and the total number of observations is 50. Find the missing frequencies f_1 and f_2.

Class	**0–20**	**20–40**	**40–60**	**60–80**	**80–100**	**100–120**
Frequency	5	f_1	10	f_2	7	8

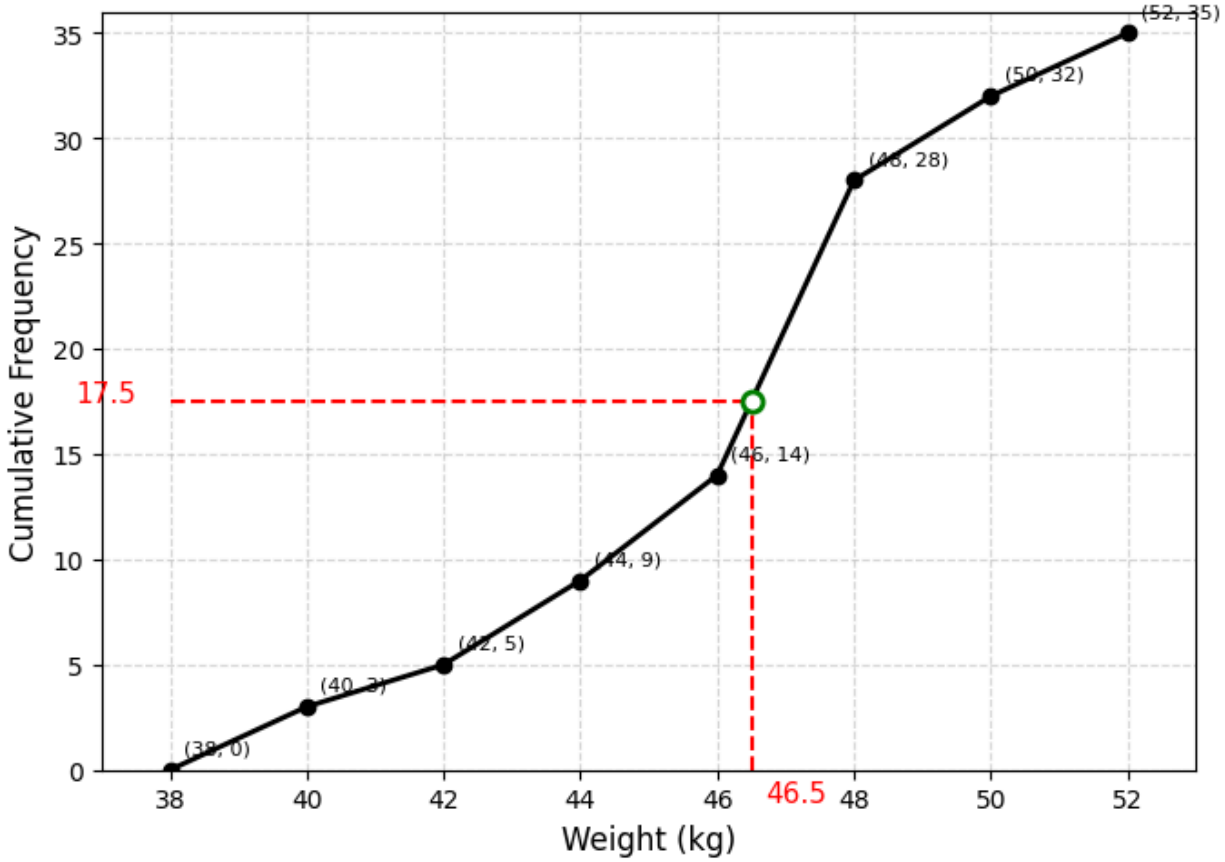
Solution:

$\Sigma f = 5 + f_1 + 10 + f_2 + 7 + 8$
$= 30 + f_1 + f_2 = 50$
$\Rightarrow f_1 + f_2 = 20$ …(i)
x_i: 10, 30, 50, 70, 90, 110
$\Rightarrow \Sigma f_i x_i = 5(10) + f_1(30) + 10(50) + f_2(70) + 7(90) + 8(110)$
$= 50 + 30f_1 + 500 + 70f_2 + 630 + 880$
$= 2060 + 30f_1 + 70f_2$
Mean = 62.8
$\Rightarrow 62.8 = (2060 + 30f_1 + 70f_2)/50$
$\Rightarrow 3140 = 2060 + 30f_1 + 70f_2$
$\Rightarrow 30f_1 + 70f_2 = 1080$
$\Rightarrow 3f_1 + 7f_2 = 108$ …(ii)
From (i): $f_1 = 20 – f_2$.
Substituting in (ii): $3(20 – f_2) + 7f_2 = 108$
$\Rightarrow 60 – 3f_2 + 7f_2 = 108$
$\Rightarrow 4f_2 = 48$
$\Rightarrow f_2 = 12$
∴ $f_1 = 8$, $f_2 = 12$

Question 6: The weights of 35 students of a class were recorded as a cumulative frequency distribution as shown below. Draw a 'less than' type ogive and find the median weight from the graph. Verify using the formula.

Weight (kg)	Less than 38	Less than 40	Less than 42	Less than 44	Less than 46	Less than 48	Less than 50	Less than 52
No. of Students	0	3	5	9	14	28	32	35

Solution:



From graph: n/2 = 35/2 = 17.5.
Draw a horizontal line at y = 17.5. Where it meets the ogive, drop a perpendicular to the x-axis. The x-value gives the median ≈ 46.5 kg.
Verification using formula:
Convert to frequency table:

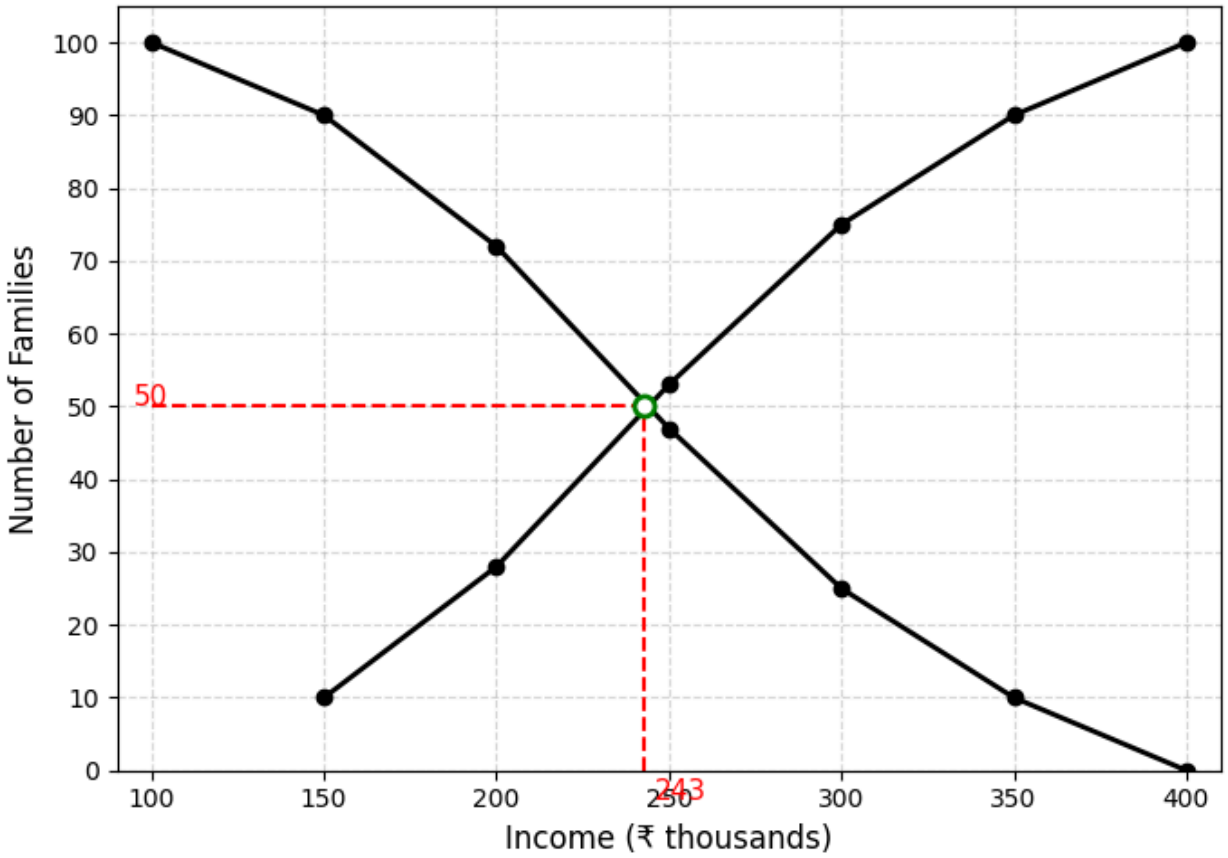
Weight (kg)	38–40	40–42	42–44	44–46	46–48	48–50	50–52
Frequency (f)	3	2	4	5	14	4	3
Cumulative Frequency (cf)	3	5	9	14	28	32	35

$n/2 = 17.5$
$\Rightarrow$ cf just ≥ 17.5 is 28
$\Rightarrow$ Median class = 46–48.
$l = 46$, $cf = 14$, $f = 14$, $h = 2$
$\Rightarrow$ Median $= 46 + [(17.5–14)/14] \times 2$
$= 46 + (3.5/14)\times 2$
$= 46 + 0.5$
$= 46.5$ kg
$\therefore$ Median Weight = 46.5 kg

Question 7: he following table gives the distribution of income of 100 families in a town. Draw 'less than' and 'more than' ogives for this data. Use the graph to find the median income. Also verify using the formula.

Income (₹ thousands)	100–150	150–200	200–250	250–300	300–350	350–400
No. of Families	10	18	25	22	15	10

Solution:



From the graph: The two ogives intersect at approx. x = 243.
Draw perpendicular to x-axis
⇒ Median ≈ ₹243 thousand.
Formula verification: n = 100, n/2 = 50
⇒ cf just ≥ 50 is 53
⇒ Median class = 200 - 250.
l = 200, cf = 28, f = 25, h = 50
⇒ Median = 200 + [(50–28)/25]×50
= 200 + (22/25)×50
= 200 + 44 = ₹244 thousand
∴ Median Income ≈ ₹243 - 244 thousand