HOTS Questions on Class 10 Maths Chapter 7 'Coordinate Geometry'

Question 1: Point A(0, 2) is equidistant from points B(3, p) and C(p, 5). Find all possible values of p.

Solution:

Given A(0, 2), B(3, p), C(p, 5). Since A is equidistant, AB = AC.

Using the Distance Formula:

$AB^2 = (3-0)^2 + (p-2)^2 = 9 + p^2 - 4p + 4 = p^2 - 4p + 13$

$AC^2 = (p-0)^2 + (5-2)^2 = p^2 + 9$

Setting $AB^2 = AC^2$:

$p^2 - 4p + 13 = p^2 + 9$

$\Rightarrow -4p = -4$

$\Rightarrow p = 1$

Question 2: Points P, Q, R, and S divide the line segment joining A(1, 2) and B(6, 7) into five equal parts. Find the coordinates of P and R.

Solution:

Five equal parts means ratios from A are 1:4, 2:3, 3:2, 4:1 for P, Q, R, S respectively.

For P (ratio 1:4 from A):

$x_P = (1\times6 + 4\times1)/(1+4) = 10/5 = 2$

$y_P = (1\times7 + 4\times2)/(1+4) = 15/5 = 3$

P = (2, 3)

For R (ratio 3:2 from A):

$x_R = (3\times6 + 2\times1)/(3+2) = 20/5 = 4$

$y_R = (3\times7 + 2\times2)/(3+2) = 25/5 = 5$

R = (4, 5)

P = (2, 3) and R = (4, 5)

Question 3: In what ratio does the point (−4, 6) divide the line segment joining A(−6, 10) and B(3, −8)?

Solution:

Let the required ratio be k:1. The dividing point is (−4, 6).

Using x-coordinate in the section formula:
$-4 = (k\times3 + 1\times(-6)) / (k+1)$
$-4(k+1) = 3k - 6$
$-4k - 4 = 3k - 6$
$-4k - 3k = -6 + 4$
$-7k = -2$
$\Rightarrow k = 2/7$
So the ratio is k:1 = 2/7 : 1 = 2:7.
The point divides AB in the ratio 2:7 internally.

Question 4: The vertices of a triangle are (1, k), (4, −3), and (−9, 7). If its area is 15 sq. units, find the value(s) of k.
Solution:
Let $(x_1,y_1) = (1,k)$, $(x_2,y_2) = (4,-3)$, $(x_3,y_3) = (-9,7)$.
Apply the area formula:
Area $= \frac{1}{2}|x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)|$
$= \frac{1}{2}|1(-3-7) + 4(7-k) + (-9)(k-(-3))|$
$= \frac{1}{2}|1(-10) + 4(7-k) + (-9)(k+3)|$
$= \frac{1}{2}|-10 + 28 - 4k - 9k - 27|$
$= \frac{1}{2}|-9 - 13k|$
Set equal to 15:
$\frac{1}{2}|-9 - 13k| = 15$ $|-9 - 13k| = 30$
Case 1: $-9 - 13k = 30 \Rightarrow -13k = 39 \Rightarrow k = -3$
Case 2: $-9 - 13k = -30 \Rightarrow -13k = -21 \Rightarrow k = 21/13$
$k = -3$ or $k = 21/13$

Question 5: Find the relation between x and y if points (2, 1), (x, y), and (7, 5) are collinear.
Solution:
For collinear points, area of triangle = 0:
$\frac{1}{2}|2(y-5) + x(5-1) + 7(1-y)| = 0$
$\Rightarrow |2y-10 + 4x + 7 - 7y| = 0$
$\Rightarrow |4x - 5y - 3| = 0$
So: $4x - 5y - 3 = 0$, which gives us $4x - 5y = 3$
Relation: $4x - 5y = 3$

Question 6: If the points A(6, 1), B(8, 2), C(9, 4), and D(p, 3) are the vertices of a parallelogram taken in order, find the value of p.

Solution:

Diagonals of a parallelogram bisect each other. Diagonals here are AC and BD.

Midpoint of AC:

$= ((6+9)/2, (1+4)/2) = (15/2, 5/2)$

Midpoint of BD:

$= ((8+p)/2, (2+3)/2) = ((8+p)/2, 5/2)$

Equating midpoints:

x: $(8+p)/2 = 15/2$

$8+p = 15$

$\Rightarrow p = 7$

Question 7: Prove that the points A(3, 0), B(6, 4), and C(−1, 3) are the vertices of a right-angled isosceles triangle.

Solution:

Compute all three sides:

$AB = \sqrt{[(6-3)^2+(4-0)^2]} = \sqrt{[9+16]} = \sqrt{25} = 5$

$BC = \sqrt{[(-1-6)^2+(3-4)^2]} = \sqrt{[49+1]} = \sqrt{50}$

$CA = \sqrt{[(3-(-1))^2+(0-3)^2]} = \sqrt{[16+9]} = \sqrt{25} = 5$

AB = CA = 5, so the triangle is isosceles.

Check the Pythagorean theorem :

$AB^2 + CA^2 = 25 + 25 = 50$

$BC^2 = 50$

Since $AB^2 + CA^2 = BC^2$, the right angle is at vertex A.

△ ABC is a right-angled isosceles triangle (right angle at A).

Question 8: Points P and Q trisect the line segment joining A(−2, 0) and B(0, 8) such that P is nearer to A. Find the area of triangle OPQ, where O is the origin.

Solution:

P divides AB in ratio 1:2 (nearer to A):

$P = ((1\times0 + 2\times(-2))/3, (1\times8 + 2\times0)/3) = ((0-4)/3, 8/3) = (-4/3, 8/3)$

Q divides AB in ratio 2:1:

$Q = ((2\times0 + 1\times(-2))/3, (2\times8 + 1\times0)/3) = (-2/3, 16/3)$

Area of △OPQ with O(0,0), P(−4/3, 8/3), Q(−2/3, 16/3):

$= \frac{1}{2}|0(8/3 - 16/3) + (-4/3)(16/3 - 0) + (-2/3)(0 - 8/3)|$

$= \frac{1}{2}|0 + (-4/3)(16/3) + (-2/3)(-8/3)|$

$= \frac{1}{2}|-64/9 + 16/9|$

$= \frac{1}{2}|-48/9|$

$= \frac{1}{2} \times 48/9 = 24/9 = 8/3$

Area of triangle OPQ = 8/3 sq. units