HOTS Questions on Class 10 Maths Chapter 8 'Introduction to Trigonometry'

Question 1: If cosec θ = 3x and cot θ = 3/x, find the value of $(x^2 - 1/x^2)$.
Solution:
Using the identity $\text{cosec}^2\theta - \cot^2\theta = 1$:
$(3x)^2 - (3/x)^2 = 1$ $9x^2 - 9/x^2 = 1$
Factor out 9:
$9(x^2 - 1/x^2) = 1$ $x^2 - 1/x^2 = 1/9$
$\Rightarrow x^2 - 1/x^2 = 1/9$

Question 2: Prove: $(\sin\theta - \cos\theta + 1) / (\sin\theta + \cos\theta - 1) = 1 / (\sec\theta - \tan\theta)$
Solution:
Take LHS: $(\sin\theta - \cos\theta + 1) / (\sin\theta + \cos\theta - 1)$
Divide numerator and denominator by cosθ:
$= (\tan\theta - 1 + \sec\theta) / (\tan\theta + 1 - \sec\theta)$
Rearrange numerator: $(\sec\theta + \tan\theta - 1)$.
Replace the 1 using the identity:
$1 = \sec^2\theta - \tan^2\theta = (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$
So: $\sec\theta + \tan\theta - 1 = \sec\theta + \tan\theta - (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$
$= (\sec\theta + \tan\theta)[1 - (\sec\theta - \tan\theta)]$
Denominator: $\tan\theta + 1 - \sec\theta = 1 - (\sec\theta - \tan\theta)$.
Let $S = \sec\theta - \tan\theta$:
LHS $= (\sec\theta + \tan\theta)(1 - S) / (1 - S) = \sec\theta + \tan\theta$
But we need $1/(\sec\theta - \tan\theta)$.
Rationalise:
$\sec\theta + \tan\theta = (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) / (\sec\theta - \tan\theta)$
$= (\sec^2\theta - \tan^2\theta) / (\sec\theta - \tan\theta) = 1 / (\sec\theta - \tan\theta)$ = RHS
LHS = RHS.
Proved.

Question 3: Prove: $(\text{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta) = 1 / (\tan\theta + \cot\theta)$
Solution:
Simplify each bracket on LHS:

$\text{cosec}\theta - \sin\theta = 1/\sin\theta - \sin\theta$
$= (1 - \sin^2\theta)/\sin\theta$
$= \cos^2\theta/\sin\theta$ $\sec\theta - \cos\theta$
$= 1/\cos\theta - \cos\theta = (1 - \cos^2\theta)/\cos\theta = \sin^2\theta/\cos\theta$
Multiply the two results:
LHS $= (\cos^2\theta/\sin\theta) \times (\sin^2\theta/\cos\theta) = \sin\theta \cdot \cos\theta$
Now simplify RHS: $1/(\tan\theta + \cot\theta)$:
$\tan\theta + \cot\theta = \sin\theta/\cos\theta + \cos\theta/\sin\theta$
$= (\sin^2\theta + \cos^2\theta)/(\sin\theta \cos\theta) = 1/(\sin\theta \cos\theta)$
So RHS $= 1 \div [1/(\sin\theta \cos\theta)] = \sin\theta \cos\theta$
LHS $= \sin\theta \cos\theta =$ RHS
Proved.

Question 4: Prove: $(\tan\theta + \sec\theta - 1) / (\tan\theta - \sec\theta + 1) = (1 + \sin\theta) / \cos\theta$
Solution:
Take LHS $= (\tan\theta + \sec\theta - 1) / (\tan\theta - \sec\theta + 1)$.
Replace 1 in numerator using $\sec^2\theta - \tan^2\theta$:
Numerator $= \tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)$
$= \tan\theta + \sec\theta - (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$
$= (\sec\theta + \tan\theta)[1 - (\sec\theta - \tan\theta)]$
$= (\sec\theta + \tan\theta)(1 - \sec\theta + \tan\theta)$
Denominator $= \tan\theta - \sec\theta + 1 = 1 - \sec\theta + \tan\theta$
Cancel $(1 - \sec\theta + \tan\theta)$:
LHS $= (\sec\theta + \tan\theta)(1 - \sec\theta + \tan\theta) / (1 - \sec\theta + \tan\theta)$
$= \sec\theta + \tan\theta = 1/\cos\theta + \sin\theta/\cos\theta$
$= (1 + \sin\theta)/\cos\theta =$ RHS
LHS $= (1 + \sin\theta)/\cos\theta =$ RHS.
Proved.

Question 5: Evaluate $\sin 18° / \cos 72° + \sqrt{3} (\tan 10° \cdot \tan 30° \cdot \tan 60° \cdot \tan 80°)$
Solution:
For the first part: $18° + 72° = 90°$, so $\sin 18° = \cos 72°$.
$\sin 18°/\cos 72° = \cos 72°/\cos 72° = 1$
For the second part: $10° + 80° = 90°$, so $\tan 80° = \cot 10°$.
$\tan 10° \times \tan 80° = \tan 10° \times \cot 10° = \tan 10° \times (1/\tan 10°) = 1$

Also, $\tan 30° = 1/\sqrt{3}$, $\tan 60° = \sqrt{3}$:
$\tan 30° \times \tan 60° = (1/\sqrt{3})(\sqrt{3}) = 1$
The product $\tan 10° \times \tan 30° \times \tan 60° \times \tan 80° = (\tan 10° \times \tan 80°) \times (\tan 30° \times \tan 60°) = 1 \times 1 = 1$
Full expression $= 1 + \sqrt{3} \times 1 = 1 + \sqrt{3}$

Question 6: If $\sin 3A = \cos(A - 26°)$, where $3A$ is an acute angle, find the value of A.
Solution:
Since $\sin X = \cos Y$ implies $X + Y = 90°$ (for acute angles):
$3A + (A - 26°) = 90°$
$4A - 26° = 90°$
$4A = 116°$
$A = 29°$

Question 7: If $x \sin^3\theta + y \cos^3\theta = \sin\theta \cos\theta$, and $x \sin\theta = y \cos\theta$, prove that: $x^2 + y^2 = 1$
Solution:
From $x \sin\theta = y \cos\theta$:
$x/y = \cos\theta/\sin\theta = \cot\theta$
So: $x = y \cos\theta/\sin\theta$
Substitute into $x \sin^3\theta + y \cos^3\theta = \sin\theta \cos\theta$:
$(y \cos\theta/\sin\theta)(\sin^3\theta) + y \cos^3\theta = \sin\theta \cos\theta$
$y \cos\theta \sin^2\theta + y \cos^3\theta = \sin\theta \cos\theta$
$y \cos\theta (\sin^2\theta + \cos^2\theta) = \sin\theta \cos\theta$
$y \cos\theta (1) = \sin\theta \cos\theta$ $y = \sin\theta$
Then $x = y \cos\theta/\sin\theta = \sin\theta \cdot \cos\theta/\sin\theta = \cos\theta$.
Therefore:
$x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$
$x^2 + y^2 = 1$.
Proved.

Question 8: If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that: $m^2 - n^2 = 4\sqrt{(mn)}$
Solution:
$m + n = 2\tan\theta$ and $m - n = 2\sin\theta$

$m^2 - n^2 = (m+n)(m-n)$:

$= (2 \tan\theta)(2 \sin\theta) = 4 \tan\theta \sin\theta$

$mn = (\tan\theta + \sin\theta)(\tan\theta - \sin\theta) = \tan^2\theta - \sin^2\theta$:

$= \sin^2\theta/\cos^2\theta - \sin^2\theta = \sin^2\theta(1/\cos^2\theta - 1) = \sin^2\theta(1 - \cos^2\theta)/\cos^2\theta = \sin^2\theta \cdot \sin^2\theta/\cos^2\theta = \sin^2\theta \tan^2\theta$

Therefore $4\sqrt{(mn)}$:

$4\sqrt{(mn)} = 4\sqrt{(\sin^2\theta \tan^2\theta)} = 4 \sin\theta \tan\theta$

So $m^2 - n^2 = 4 \tan\theta \sin\theta = 4 \sin\theta \tan\theta = 4\sqrt{(mn)}$