# HOTS Questions on Class 9 Maths Chapter 2 Introduction to Linear Polynomials

## HOTS Practice Questions on Linear Polynomials

### Advanced HOTS Questions

**Q1.** If p(x) = ax + b and q(x) = bx + a, and both have the same zero, prove that a = b or a = −b.

**Q2.** A linear polynomial p(x) is such that p(1) = p(−1). What conclusion can you draw? Is p(x) linear? Explain.

**Q3.** The sum of the zero of p(x) = 3x + k and the zero of q(x) = kx + 3 is zero. Find k.

### Multi-Step Reasoning Questions

**Q4.** p(x) = $(a^2 - 9)x + (a - 3)$ is a polynomial. (i) For what values of a is it linear? (ii) For what values is the zero x = 1?

**Q5.** The zero of p(x) = 2x + 1 is α. Without finding α explicitly, find the zero of p(3x − 2) in terms of α.

### Challenge Problems

**Q6.** A linear polynomial p(x) satisfies p(0) + p(1) + p(2) = 12 and p(3) = 10. Find p(x).

**Q7.** If the zero of $(a + b)x + (a - b)$ is the same as the zero of $(2a - 1)x + (a + b)$, find the relationship between a and b.

### Answer Explanations

**A1.** Zero of p(x) = −b/a. Zero of q(x) = −a/b.

Set equal: $-b/a = -a/b$, $b^2 = a^2$, $a^2 - b^2 = 0$, $(a+b)(a-b) = 0$. So a = b or a = −b.

**A2.** p(1) = a + b. p(−1) = −a + b. Set equal: a + b = −a + b, 2a = 0, a = 0. But if a = 0, the degree of p(x) = ax + b drops to 0. it becomes a constant polynomial. Therefore, p(x) cannot be linear if p(1) = p(−1). Any linear polynomial has different values at x = 1 and

$x = -1$.

**A3.** Zero of $p(x) = -k/3$. Zero of $q(x) = -3/k$ (assuming $k \neq 0$).

Sum $= -k/3 + (-3/k) = 0$, $-k/3 = 3/k$, $-k^2 = 9$, $k^2 = -9$. No real solution.

If $k = 0$: $q(x) = 3$ (constant, no zero). So there is no real value of k that satisfies the condition this is an important insight into the limits of algebraic manipulation.

**A4.** (i) For linear: $a^2 - 9 \neq 0 \rightarrow a \neq 3$ and $a \neq -3$. $a \in \mathbb{R}$, $a \neq \pm 3$.

(ii) Zero at $x = 1$: $(a^2 - 9)(1) + (a - 3) = 0$

$a^2 - 9 + a - 3 = 0$

$a^2 + a - 12 = 0$

$(a+4)(a-3) = 0$

$a = -4$ or $a = 3$.

But $a \neq 3$ (from condition i), so $a = -4$.

Verify: $(16-9)x + (-4-3) = 7x - 7$.

Zero: $7x - 7 = 0$, $x = 1$ .

**A5.** Zero of $p(x) = 2x + 1$ is $\alpha$, meaning $2\alpha + 1 = 0$.

Find zero of $p(3x - 2)$: replace x with $(3x - 2)$ in $p(x)$: $p(3x-2) = 2(3x-2) + 1$

$= 6x - 4 + 1$

$= 6x - 3$.

Zero: $6x - 3 = 0 \rightarrow x = ½$.

Express in terms of $\alpha$: from $2\alpha + 1 = 0$, $\alpha = -½$. So $½ = -\alpha$.

Zero of $p(3x-2)$ is $-\alpha$.

**A6.** Let $p(x) = ax + b$.

$p(0) + p(1) + p(2) = b + (a+b) + (2a+b) = 3a + 3b = 12$, $a + b = 4$ … (i).

$p(3) = 3a + b = 10$ … (ii).

Subtract (i) from (ii): $2a = 6$, $a = 3$. Then $b = 1$. $p(x) = 3x + 1$.

Verify: $p(0) + p(1) + p(2) = 1 + 4 + 7 = 12$.

$p(3) = 10$.

**A7.** Zero of first: $-(a - b)/(a + b)$. Zero of second: $-(a + b)/(2a - 1)$.

Set equal: $-(a - b)/(a + b) = -(a + b)/(2a - 1)$.

Cross multiply: $(a - b)(2a - 1) = (a + b)^2$.

Expand left: $2a^2 - a - 2ab + b$. Expand right: $a^2 + 2ab + b^2$.

$2a^2 - a - 2ab + b = a^2 + 2ab + b^2$.

$a^2 - 4ab - b^2 - a + b = 0$.

This is the required relationship between a and b.