HOTS Questions on Class 9 Maths Chapter 5 I'm Up and Down, and Round and Round

Shape Interpretation Questions

Question 1: Two identical circles of radius 7 cm overlap such that the centre of each circle lies on the boundary of the other circle. Describe the shape formed in the overlapping region. What are the angles at the vertices of this overlapping shape?

Question 2: A square with side 14 cm has a circle drawn inside it, touching all four sides. A second circle is drawn inside the same square but touching only two opposite sides and passing through the midpoints of the other two sides. Compare the areas of the two circles. (Use $\pi = 22/7$)

Question 3: A circular disc is placed flat on a table. A second disc of the same size is placed on top of the first, rotated by 30°. If you look from directly above, what shape do you see at the region where they overlap but do not perfectly align? Describe the visual appearance.

Diagram-Based HOTS Problems

Question 4: Draw a circle with centre O and radius 6 cm. Mark two points A and B on the circle such that the arc AB subtends an angle of 60° at the centre. Draw the chord AB. Is triangle OAB equilateral? Justify your answer.

Question 5: A semicircle is drawn on the diameter AB of length 20 cm. A point P is marked on the curved part of the semicircle. What is angle APB? If P moves along the curved part of the semicircle, does the angle APB change? Justify your answer using geometry.

Answer Explanations

Answer 1: When two circles of equal radius (each 7 cm) overlap so that the centre of each lies on the boundary of the other, the overlapping region is a shape bounded by two circular arcs. This shape is called a vesica piscis or lens shape. Each arc is a portion of one circle.

Because the radius of each circle equals the distance between the two centres (both are 7 cm the centre of circle 1 lies on circle 2, so the distance between centres

equals the radius), triangle formed by the two centres and either intersection point is equilateral. Therefore, each arc subtends an angle of 60° at its own centre, and the angles at the two pointed tips (vertices) of the lens shape are each 60°.

Answer 2: Circle inscribed in a square of side 14 cm touches all four sides. Its diameter equals the side of the square.

Radius of first circle = 14 ÷ 2 = 7 cm.

Area of first circle = $\pi \times 7^2 = (22/7) \times 49 = 154$ cm².

Second circle touches two opposite sides (say top and bottom), passing through the midpoints of the other two sides. Its diameter = 14 cm (same as before) so it is actually the same size.

Both circles have the same radius of 7 cm and therefore the same area of 154 cm². They are simply oriented differently within the square. This is an interesting result that requires spatial reasoning to see: a circle that touches two opposite sides and passes through the midpoints of the other two sides in a square has the exact same radius as the inscribed circle.

Answer 3: When two identical circular discs overlap perfectly (zero rotation), the overlapping region is the entire disc a full circle. As one disc is rotated by 30°, the region where they overlap perfectly is reduced. The overlap region when one disc is rotated relative to the other is a lens-shaped (vesica-like) region bounded by two arcs, one from each disc. The more the rotation increases toward 180°, the thinner the lens shape becomes. At 30° rotation, you would see a relatively large lens-shaped overlap region, with the two outer crescents of the discs extending beyond the overlap. The visual impression from directly above is a circle with two curved "wing" projections on opposite sides.

Answer 4: Draw the circle with centre O and radius 6 cm. Angle AOB = 60° (given). OA and OB are both radii, so OA = OB = 6 cm.

Since OA = OB (both radii), triangle OAB is isosceles.

The angle at the centre, angle AOB = 60°. Since OA = OB, the base angles of the isosceles triangle are equal. So angle OAB = angle OBA.

Angle OAB + angle OBA + 60° = 180°

2 × angle OAB = 120°

angle OAB = 60°.

All three angles of triangle OAB are 60°, and all three sides are equal (each = 6 cm).

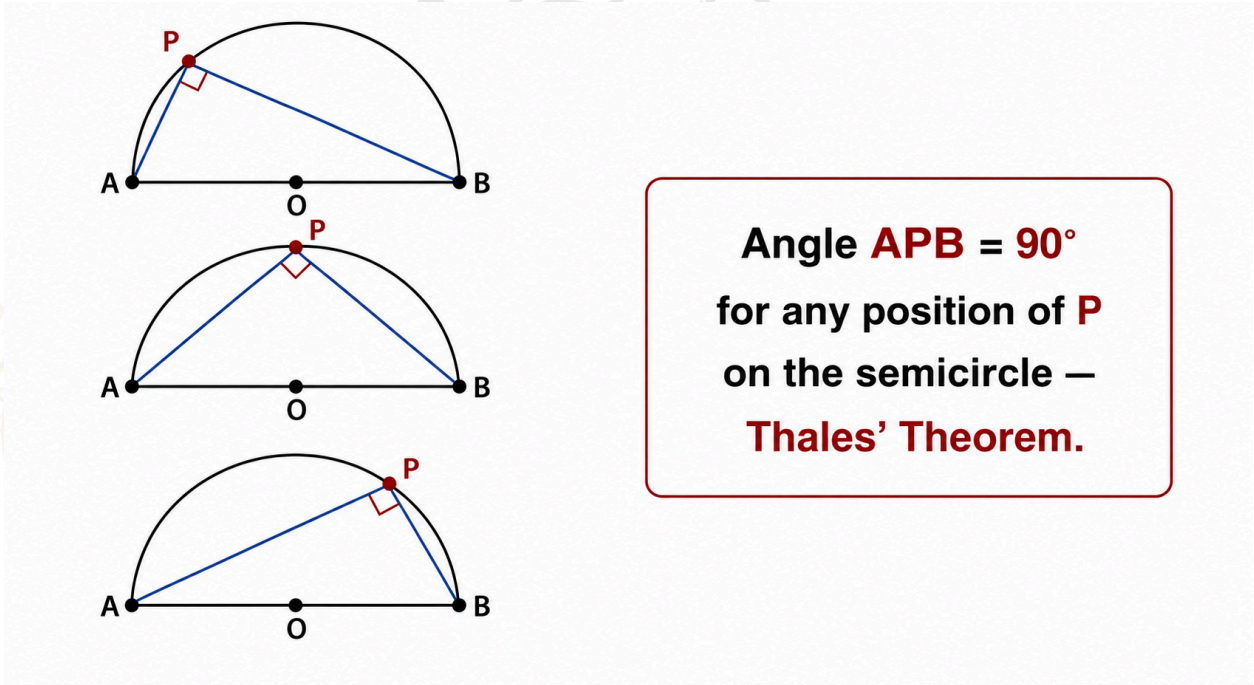
Yes, triangle OAB is equilateral. This is a key geometric result: when two radii of a circle form a 60° angle at the centre, the chord connecting them equals the radius, forming an equilateral triangle.

Answer 5: Angle APB = 90° for every position of P on the semicircle.

This follows directly from Thales' Theorem, which states that any angle inscribed in a semicircle (that is, an angle whose vertex lies on the curved part of a semicircle and whose two sides pass through the endpoints of the diameter) is always a right angle.

As P moves along the semicircle, the angle APB does NOT change it remains 90° at every position. This is because Thales' Theorem applies to every point on the semicircle equally, regardless of where P is located on the curved arc.

This result is sometimes called the angle in a semicircle theorem, and it is one of the most important and elegant results in all of circle geometry.



Mixed HOTS Practice Questions on Circular Geometry

Advanced HOTS Questions

Question 1: A circular pond has a circumference of 176 metres. A gardener plants trees at equal intervals of 8 metres along the boundary starting from a fixed point. How many trees does the gardener plant? After planting all the trees, the gardener walks around the boundary once to water each tree. How far does she walk in total? (Use π = 22/7)

Question 2: A circular wheel of radius 49 cm is placed at the bottom of a hill. It rolls up the hill until it has made exactly 50 rotations. What distance along the slope has the wheel covered? (Use π = 22/7)

Question 3: The wheels of two bicycles have radii in the ratio 3 : 4. If both bicycles travel the same distance, what is the ratio of the number of rotations made by their wheels?

Challenge Problems

Question 4: A goat is tied to a peg at the corner of a square field with side 14 metres. The rope is 7 metres long. What area of grass can the goat graze if it can only graze outside the square? (Use π = 22/7)

Question 5: A circular track of radius 100 metres has two runners starting from the same point at the same time. Runner A runs at 5 metres per second and Runner B runs at 8 metres per second, both in the same direction. How long will it take before they meet again at the starting point for the first time? (Use π = 22/7)

Multi-Concept Questions

Question 6: A semicircular piece of wire is straightened out and then bent into a full circle. If the radius of the semicircle was 14 cm, what is the radius of the new circle formed? Compare the areas of the semicircle and the new full circle. (Use π = 22/7)

Answer 1: Circumference = 176 m.

Number of trees = Circumference ÷ interval = 176 ÷ 8 = 22 trees.

Walking distance = circumference = 176 metres (one full loop).

Answer 2: Radius = 49 cm. Circumference = 2 × (22/7) × 49 = 308 cm.

Distance for 50 rotations = 50 × 308 = 15,400 cm = 154 metres along the slope.

Answer 3: Distance = Number of rotations × Circumference.

Since distance is the same for both:

$\text{Rotations}_1 \times 2\pi r_1 = \text{Rotations}_2 \times 2\pi r_2$

$\text{Rotations}_1 / \text{Rotations}_2 = r_2 / r_1 = 4/3$.

Ratio of rotations of bicycle 1 to bicycle 2 = 4 : 3.

The bicycle with the smaller wheel (ratio 3) makes more rotations to cover the same distance.

Answer 4: The goat is at a corner of the square. The rope is 7 metres. Outside the square, the goat can swing around the corner in a quarter-circle arc (90° sector) because the square's corner has an interior angle of 90°, leaving 360° − 90° = 270° of potential swing outside, but the square's two adjacent sides limit this to a quarter circle on the outside.

Wait since the goat is at a corner with a 90° internal angle, the external angle is 360° − 90° = 270°. But the two walls of the square at the corner block movement, so the goat grazes a 270° sector of radius 7 m outside the square corner, and potentially two smaller quarter-circle sectors at adjacent corners.

For this HOTS question, considering just the main arc (270° sector):

Area = $(270/360) \times \pi \times 7^2 = (3/4) \times (22/7) \times 49 = (3/4) \times 154 = 115.5\ m^2$

Answer 5: Circumference of track = 2 × (22/7) × 100 = 2 × 314.29 = 628.57 metres approximately.

They meet again when the faster runner has lapped the slower runner by exactly one full circumference.

Relative speed = 8 − 5 = 3 m/s.

Time = Circumference ÷ Relative speed = 628.57 ÷ 3 ≈ 209.5 seconds (approximately 3 minutes 30 seconds).

Using exact value: circumference = 2 × (22/7) × 100 = 4400/7 m. Time = (4400/7) ÷ 3 = 4400/21 ≈ 209.52 seconds.

Answer 6: Length of wire = Length of semicircle (arc only) = $(1/2) \times 2\pi r = \pi r = (22/7) \times 14 = 44$ cm.

This wire is bent into a full circle: $2\pi R = 44$.

$R = 44 \div (2 \times 22/7) = 44 \times 7/44 = 7$ cm.

Area of semicircle (original) = $(1/2) \times \pi r^2 = (1/2) \times (22/7) \times 14^2 = (1/2) \times 616 = 308$ cm².

Area of new circle = $\pi R^2 = (22/7) \times 7^2 = 154$ cm².

The new circle (154 cm²) has less area than the original semicircle (308 cm²), even though the same length of wire was used. This is because the semicircle's straight diameter edge (28 cm) was not included in the wire length only the curved arc was. The full semicircle shape was larger because it also enclosed the diameter.

ORCHIDS® The International School
Indian Education, International Approach