HOTS Questions on Class 9 Maths Chapter 8 Predicting What Comes Next: Exploring Sequences and Progressions

HOTS Questions Involving Visual and Geometric Patterns

Q1. Equilateral triangles are built from matchsticks in a row: 1 triangle needs 3 sticks, 2 triangles need 5 sticks, 3 need 7, 4 need 9… How many sticks are needed for 20 triangles?

Ans. Sequence: 3, 5, 7, 9… AP with a = 3, d = 2.

$a_{20} = 3 + 19(2) = 3 + 38 = 41$ matchsticks

Q2. A pattern of squares is built using dots as corners. Figure 1 has 4 dots, Figure 2 has 9, Figure 3 has 16, Figure 4 has 25. What is the rule? How many dots in Figure 10?

Ans. $4 = 2^2$, $9 = 3^2$, $16 = 4^2$, $25 = 5$…

Figure n has $(n + 1)^2$ dots.

Figure 10 has $(10 + 1)^2 = 11^2 = 121$ dots

Q3. Tiles are arranged in an L-shape. Stage 1 uses 3 tiles, Stage 2 uses 5, Stage 3 uses 7. At what stage will the designer first need more than 100 tiles?

Ans. AP: a = 3, d = 2. $a_n = 3 + (n-1)(2) = 2n + 1$.

Set $2n + 1 > 100 \rightarrow 2n > 99 \rightarrow n > 49.5 \rightarrow n = 50$

Stage 50: $a_{50} = 2(50) + 1 = 101$ tiles (first stage exceeding 100).

Q4. A square is divided into smaller squares. Stage 1: 1 square. Stage 2: 4 small squares. Stage 3: 9 small squares. Stage 4: 16. Is the difference between consecutive stages an AP? Find the rule.

Ans. Differences: $4-1=3$, $9-4=5$, $16-9=7$.

Differences are 3, 5, 7… which form an AP with $d = 2$.

The original sequence 1, 4, 9, 16 is not an AP but a quadratic sequence (n^2).

Rule: Stage n has n^2 small squares.

Q5. A pattern of pentagons made from matchsticks: Pentagon 1 uses 5 sticks, Pentagon 2 uses 9, Pentagon 3 uses 13… Find the rule and how many sticks are needed for Pentagon 15.

Ans. AP: $a = 5$, $d = 4$. $a_n = 5 + (n-1)(4) = 4n + 1$.

$a_{15} = 4(15) + 1 = 60 + 1 = 61$ matchsticks

HOTS Questions Based on Comparing Different Progressions

Q1. AP P: 3, 7, 11, 15… and AP Q: 63, 65, 67, 69… Do these two APs share any common term? Find the first common term.

Ans. nth term of P: $3 + (n-1)(4) = 4n - 1$

mth term of Q: $63 + (m-1)(2) = 2m + 61$

Set equal: $4n - 1 = 2m + 61 \rightarrow 4n - 2m = 62 \rightarrow 2n - m = 31 \rightarrow m = 2n - 31$

For the smallest positive integer n where m is also a positive integer:

n must be > 15.5, so try $n = 16$: $m = 2(16) - 31 = 1$.

When $n = 16$: term of P $= 4(16)-1 = 63$. When $m = 1$: term of Q $= 63$

First common term = 63

Q2. AP A has first term 2 and common difference 3. AP B has first term 3 and common difference 2. Compare the 10th terms and explain which grows faster and why.

Ans.

$A_{10} = 2 + 9(3) = 2 + 27 = 29$

$B_{10} = 3 + 9(2) = 3 + 18 = 21$

AP A grows faster because it has a larger common difference (d = 3 vs d = 2). The common difference determines the rate of growth the larger the d, the faster the sequence increases.

AP A overtakes AP B even though B started with a slightly larger first term, the higher growth rate of A quickly surpasses it.

Q3. If the nth term of one AP is (4n + 1) and the nth term of a second AP is (6n − 3), for what value of n do they have equal terms?

Ans. Set equal: $4n + 1 = 6n - 3$, $-2n = -4$, $n = 2$

At n = 2: First AP term = 4(2)+1 = 9. Second AP term = 6(2)−3 = 9

The two APs have equal terms at n = 2, with value 9.

Q4. A student says: "If two APs have the same common difference, they will eventually have a common term." Is this always true? Give a reason or counterexample.

Ans. Not always. Consider AP A: 1, 3, 5, 7… (odd numbers, d = 2) and AP B: 2, 4, 6, 8… (even numbers, d = 2).

Both have d = 2. A consists entirely of odd numbers and B consists entirely of even numbers. They share no common term, because odd and even numbers never coincide.

Two APs with the same d will share a common term only if their first terms have the same remainder when divided by d (i.e., $a_1 \equiv b_1 \bmod d$). If the remainders differ, they never share a common term.

Q5. An AP has first term a and common difference d. A new AP is formed by taking every alternate term of the original: $a_1, a_3, a_5, a_7\ldots$ What are the first term and common difference of the new AP?

Ans. Original AP: $a_1 = a$, $a_3 = a + 2d$, $a_5 = a + 4d$, $a_7 = a + 6d\ldots$

New AP terms: $a, a + 2d, a + 4d, a + 6d\ldots$

First term = a (unchanged)

New common difference = $(a + 2d) - a = 2d$

The new AP has the same first term but double the common difference. This is a standard result: taking every kth term of an AP gives a new AP with common difference kd.

Mixed HOTS Practice Questions on Sequences and Progressions

Q1. The sum of three consecutive terms of an AP is 27 and their product is 504. Find the three terms.

Ans. Let the three terms be $(a - d), a, (a + d)$.

Sum: $(a - d) + a + (a + d) = 3a = 27$, $a = 9$

Product: $(9 - d)(9)(9 + d) = 504$, $9(81 - d^2) = 504$, $81 - d^2 = 56$, $d^2 = 25$, $d = \pm 5$

If $d = 5$: terms are 4, 9, 14

If $d = -5$: terms are 14, 9, 4 (same set, reversed)

Q2. In an AP, the pth term is q and the qth term is p. Find the (p + q)th term.

Ans. a□ = a + (p − 1)d = q … (i)

a_φ = a + (q − 1)d = p … (ii)

Subtract (ii) from (i): (p − q)d = q−p, d(p − q) = −(p − q), d = −1 (assuming p ≠ q)

From (i): a + (p−1)(−1) = q, a = q + p − 1 = (p + q − 1)

a_(p + q) = (p + q − 1) + (p + q − 1)(− 1) = (p + q − 1)(1 − 1) = 0

The (p + q)th term is always zero.

Q3. An AP has n terms. The first term is 1 and the last term is 99. The common difference is 2. Find n, and also determine whether 50 is a term of this AP.

Ans. a = 1, d = 2, a□ = 99.

1 + (n−1)(2) = 99, (n−1)(2) = 98, n − 1 = 49, n = 50

Check if 50 is a term: 1 + (k−1)(2) = 50, (k−1)(2) = 49, k−1 = 24.5 → k is not a whole number.

50 is not a term of this AP (the AP contains only odd numbers: 1, 3, 5, … 99).

Q4. The kth term of a sequence is a□ = $2k^2$ − k. (i) Find a_1, a_2, a_3, a_4. (ii) Is this an AP? (iii) Find the difference between consecutive terms and describe the pattern.

Ans.

a_1 = 2(1) − 1 = 1 a_2 = 2(4) − 2 = 6 a_3 = 2(9) − 3 = 15 a_4 = 2(16) − 4 = 28

(ii) Differences: 6 − 1 = 5, 15 − 6 = 9, 28 − 15 = 13. Not constant → Not an AP.

(iii) The differences are 5, 9, 13… which form an AP with d = 4. This is a quadratic sequence its first differences form an AP, not the sequence itself.

Q5. Two APs start at different points but have their 5th terms equal. AP X has d = 3. AP Y has d = 5. Find the relationship between their first terms.

Ans. 5th term of X = $a_x + 4(3) = a_x + 12$

5th term of Y = $a_Y + 4(5) = a_Y + 20$

Set equal: $a_x + 12 = a_Y + 20$, $a_x - a_Y = 8$, $a_x = a_Y + 8$

The first term of AP X must be 8 more than the first term of AP Y for their 5th terms to be equal.

Q6. The nth term of a sequence is $a_n = n/(n+1)$. (i) Is this a polynomial-type sequence? (ii) Find the difference $a_5 - a_4$. (iii) As n becomes very large, what does a_n approach?

Ans.

(i) $a_n = n/(n+1)$ involves division not a polynomial. It is a rational sequence.

(ii) $a_5 = 5/6$, $a_4 = 4/5$.

$a_5 - a_4 = 5/6 - 4/5 = 25/30 - 24/30 = 1/30$

(iii) As $n \to \infty$: $n/(n+1) = 1/(1 + 1/n) \to 1/(1+0) = 1$

The sequence approaches 1 but never reaches it.

Q7. An AP has its 4th, 7th, and 10th terms in the ratio 1: 2 : 3. Find the common difference as a multiple of the first term.

Ans. Let first term = a, common difference = d.

$a_4 = a + 3d$, $a_7 = a + 6d$, $a_{10} = a + 9d$

Given ratio 1:2:3:

$(a + 3d)/1 = (a + 6d)/2 = (a + 9d)/3$

From first two: $2(a + 3d) = a + 6d \to 2a + 6d = a + 6d \to a = 0$

Check: if a = 0, terms are 3d, 6d, 9d in ratio 1:2:3

So a = 0 and d can be any value. The common difference as a multiple of a is undefined (since a = 0), but the terms are always in ratio 1:2:3 regardless of d.