Real Numbers is the first chapter of Class 10 Maths and one of the most important topics for your CBSE board exam. This chapter covers everything from Euclid's Division Lemma to the Fundamental Theorem of Arithmetic, rational and irrational numbers, HCF, LCM, and decimal expansions. The solutions are prepared to help students understand the concepts clearly and perform better in school and board exams. A free PDF is also available for easy offline practice and quick revision.

Real Numbers Questions and Answers

Question 1: What are real numbers? Give two examples each of rational and irrational numbers.

Answer:

Real numbers are the collection of all rational and irrational numbers. Every point on the number line represents a real number.

Rational number examples:

3/4 = 0.75 (terminating decimal)

-2/5 = -0.4 (terminating decimal)

Irrational number examples:

$\sqrt{2}$ = 1.41421... (non-terminating, non-recurring)

π = 3.14159... (non-terminating, non-recurring)

Real numbers = Rational $\cup$ Irrational

Question 2: Show that $\sqrt{2}$ is irrational.

Answer:

Proof by contradiction:

Assume $\sqrt{2}$ is rational. Then $\sqrt{2} = p/q$ where p and q are integers with no common factor (HCF = 1).

Squaring both sides:

$2 = p^2/q^2$

$p^2 = 2q^2$ ...(1)

This means p^2 is even, so p is even.

Let p = 2m for some integer m.

Substituting in (1):

$(2m)^2 = 2q^2$

$4m^2 = 2q^2$

$q^2 = 2m^2$

This means q^2 is even, so q is even.

But if both p and q are even, their HCF ≠ 1. This contradicts our assumption.

Therefore √2 is irrational.

Question 3: Classify the following as rational or irrational: √4, √7, 22/7, π, 0.1

Answer:

√4 = 2 = 2/1 → Rational

√7 = 2.6457... (non-terminating, non-recurring) → Irrational

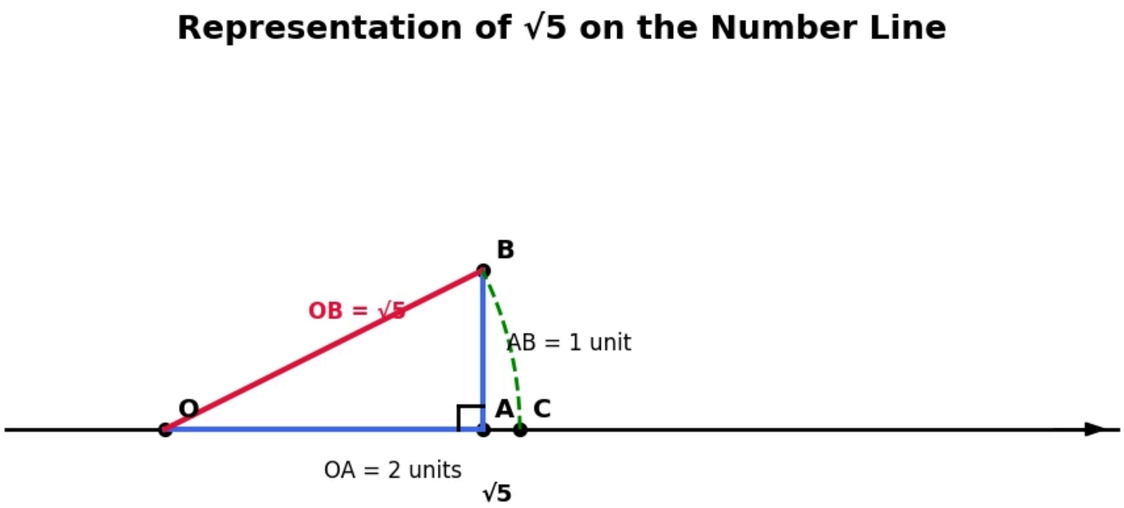
22/7 = 3.142857... (non-terminating but recurring) → Rational

π = 3.14159... (non-terminating, non-recurring) → Irrational

0.1 = 0.111... = 1/9 → Rational

Many students confuse 22/7 with π. They are not equal 22/7 is rational, π is irrational.

Question 4: Represent √5 on the number line.



Answer:

Step 1: Draw a number line

Step 2: Mark point O at 0 and A at 2

Step 3: At A, draw perpendicular AB of length 1 unit

Step 4: Join OB

Using Pythagoras:

$OB^2 = OA^2 + AB^2 = 4 + 1 = 5$

$OB = \sqrt{5}$

Step 5: With O as center and radius OB,

draw an arc cutting the number line at C

Point C represents √5 on the number line.

Question 5: State Euclid's Division Lemma. Find q and r when a = 56 and b = 9.

Answer:

Euclid's Division Lemma: For any two positive integers a and b, there exist unique integers q and r such that:

$a = bq + r$, where $0 \leq r < b$

Finding q and r for a = 56, b = 9:

$56 \div 9 = 6$ remainder 2

So: $56 = 9 \times 6 + 2$

q = 6, r = 2

Verification: $9 \times 6 + 2 = 54 + 2 = 56$

Question 6: Use Euclid's Division Algorithm to find HCF of 870 and 225.

Answer:

Step-by-step using Euclid's Algorithm:

Step 1: Apply lemma to 870 and 225

$870 = 225 \times 3 + 195$

Step 2: Apply lemma to 225 and 195

$225 = 195 \times 1 + 30$

Step 3: Apply lemma to 195 and 30

$195 = 30 \times 6 + 15$

Step 4: Apply lemma to 30 and 15

$30 = 15 \times 2 + 0$

Remainder = 0,

The last non-zero remainder is 15.

HCF(870, 225) = 15

Verification:

$870 = 15 \times 58$

$225 = 15 \times 15$

Question 7: Find HCF of 135 and 225 using Euclid's Division Algorithm.

Answer:

Since 225 > 135:

$225 = 135 \times 1 + 90$

Apply to 135 and 90:

$135 = 90 \times 1 + 45$

Apply to 90 and 45:

$90 = 45 \times 2 + 0$

Remainder = 0,

HCF(135, 225) = 45

Question 8: Show that every positive even integer is of the form 2q and every positive odd integer is of the form 2q + 1.

Answer: Let n be any positive integer. By Euclid's Lemma with b = 2:

$n = 2q + r$, where $0 \le r < 2$

So r = 0 or r = 1

Case 1: $r = 0 \rightarrow n = 2q$ (even number)

Case 2: $r = 1 \rightarrow n = 2q + 1$ (odd number)

Since every integer is either even or odd:

- Every even integer = 2q
- Every odd integer = 2q + 1

Question 9: Prove that any positive integer is of the form 3q, 3q+1, or 3q+2.

Answer:

Let n be any positive integer. By Euclid's Lemma with b = 3:

$n = 3q + r$, where $0 \le r < 3$

Possible values of r: 0, 1, 2

Case 1: $r = 0 \rightarrow n = 3q$

Case 2: $r = 1 \rightarrow n = 3q + 1$

Case 3: $r = 2 \rightarrow n = 3q + 2$

Therefore any positive integer must be of the form 3q, 3q+1, or 3q+2.

Question 10: Express 156 as a product of prime factors.

Answer:

Using prime factorisation (factor tree method):

$156 \div 2 = 78$

$78 \div 2 = 39$

$39 \div 3 = 13$

13 is prime

$156 = 2 \times 2 \times 3 \times 13$

$156 = 2^2 \times 3 \times 13$

Verification: $2^2 \times 3 \times 13 = 4 \times 3 \times 13 = 4 \times 39 = 156$

Question 11: Find HCF and LCM of 6, 72, and 120 using prime factorisation method.

Answer:

Step 1: Prime factorisation

$6 = 2 \times 3$

$72 = 2^3 \times 3^2$

$120 = 2^3 \times 3 \times 5$

Step 2: Find HCF

HCF = Product of lowest powers of common primes

$HCF = 2^1 \times 3^1 = 6$

Step 3: Find LCM

LCM = Product of highest powers of all primes

$LCM = 2^3 \times 3^2 \times 5 = 8 \times 9 \times 5 = 360$

Answer: HCF = 6, LCM = 360

Question 12: The HCF of 306 and 657 is 9. Find their LCM.

Answer:

Using the formula:

HCF × LCM = Product of two numbers

9 × LCM = 306 × 657

9 × LCM = 201,042

LCM = 201,042 ÷ 9

LCM = 22,338

Answer: LCM = 22,338

Question 13: Check whether 6^n can end with the digit 0 for any natural number n.

Answer:

For a number to end with digit 0, it must be divisible by 10, which means it must have both 2 and 5 as prime factors.

Prime factorisation of 6^n:

$6^n = (2 \times 3)^n = 2^n \times 3^n$

6^n contains only prime factors 2 and 3.

It does NOT contain 5 as a factor.

Therefore 6^n can never end with digit 0 for any natural number n.

Question 14: Without actual division, determine if 13/3125 has a terminating decimal expansion. If yes, find it.

Answer:

Check the denominator:

$3125 = 5^5$

Denominator $= 5^5 = 2^0 \times 5^5$

Since the denominator has only factors of 2 and 5, it is a terminating decimal.

Finding the decimal:

$13/3125 = 13/5^5$

Multiply numerator and denominator by 2^5:

$= (13 \times 32)/(5^5 \times 2^5)$

= 416/100000

= 0.00416

Answer: 13/3125 = 0.00416 (terminating)

Question 15: Write the condition for p/q to have a terminating decimal expansion.

Answer:

A rational number p/q (in lowest terms, HCF(p,q) = 1) has a terminating decimal expansion if and only if:

The denominator q is of the form $2^n \times 5^m$

Where n and m are non-negative integers (0, 1, 2, 3...)

Examples:

$7/8 = 7/2^3$ → Terminating (0.875)

3/6 = 1/2 → Terminating (0.5)

1/7 → 7 is not of form $2^n \times 5^m$ → Non-terminating

Question 16: Classify the following as terminating or non-terminating recurring:

$17/8$, $15/1600$, $29/343$, $23/2^3 \times 5^2$

Answer:

17/8:

$8 = 2^3$ (only factor of 2)

Terminating

17/8 = 2.125

15/1600:

$1600 = 2^6 \times 5^2$

Terminating

29/343:

$343 = 7^3$ (contains 7, not of $2^n \times 5^m$ form)

Non-terminating recurring

$23/(2^3 \times 5^2)$:

Denominator = $2^3 \times 5^2$

Terminating

Question 17: Express $0.\overline{6}$ as a fraction in simplest form.

Answer:

Let x = 0.6 = 0.6666...

Multiply both sides by 10:

10x = 6.6666... = 6.6

Subtract original:

10x - x = 6.6 - 0.6

9x = 6

x = 6/9 = 2/3

Answer: 0.6 = 2/3

Question 18: Express $0.4\overline{7}$ as a fraction.

Answer:

Let x = 0.47 = 0.4777...

Multiply by 10:

10x = 4.777... = 4.7 ...(1)

Multiply by 100:

100x = 47.777... = 47.7 ...(2)

Subtract (1) from (2):

90x = 43

x = 43/90

Answer: 0.47 = 43/90

Question 19: Verify the closure property of irrational numbers under addition with an example showing it does NOT always hold.

Answer:

Claim: Sum of two irrational numbers is NOT always irrational.

Counterexample:

Let a = $\sqrt{3}$ and b = $-\sqrt{3}$

Both $\sqrt{3}$ and $-\sqrt{3}$ are irrational numbers.

$a + b = \sqrt{3} + (-\sqrt{3}) = 0$

But 0 is a rational number.

Therefore irrational numbers are NOT closed under addition. The sum of two irrational numbers can be rational.

Example where sum IS irrational:

$\sqrt{2} + \sqrt{3} = 2.414... + 1.732... =$ irrational

Question 20: State and verify the distributive property with real numbers. Use a = 2, b = $\sqrt{3}$, c = $\sqrt{5}$.

Answer:

Distributive Property:

$a \times (b + c) = (a \times b) + (a \times c)$

Verification:

LHS = $2 \times (\sqrt{3} + \sqrt{5})$

$= 2\sqrt{3} + 2\sqrt{5}$

RHS = $(2 \times \sqrt{3}) + (2 \times \sqrt{5})$

$= 2\sqrt{3} + 2\sqrt{5}$

LHS = RHS

Distributive property holds for real numbers.

Question 21: Rationalise the denominator of $1/(\sqrt{7} - \sqrt{6})$.

Answer:

Multiply numerator and denominator by the conjugate $(\sqrt{7} + \sqrt{6})$:

$= 1/(\sqrt{7} - \sqrt{6}) \times (\sqrt{7} + \sqrt{6})/(\sqrt{7} + \sqrt{6})$

$= (\sqrt{7} + \sqrt{6})/((\sqrt{7})^2 - (\sqrt{6})^2)$

$= (\sqrt{7} + \sqrt{6})/(7 - 6)$

$= (\sqrt{7} + \sqrt{6})/1$

$= \sqrt{7} + \sqrt{6}$

Answer: $1/(\sqrt{7} - \sqrt{6}) = \sqrt{7} + \sqrt{6}$

Question 22: Simplify: $(\sqrt{5} + \sqrt{2})/(\sqrt{5} - \sqrt{2})$

Answer:

Multiply by conjugate $(\sqrt{5} + \sqrt{2})$:

$= (\sqrt{5} + \sqrt{2})^2 / ((\sqrt{5})^2 - (\sqrt{2})^2)$

$= (5 + 2\sqrt{10} + 2) / (5 - 2)$

$= (7 + 2\sqrt{10}) / 3$

Answer: $(7 + 2\sqrt{10})/3$

Question 23: State the theorem on decimal expansion of rational numbers. Classify 35/50 and 17/6.

Answer:

Theorem: Let $x = p/q$ be a rational number where HCF(p,q) = 1. Then:

- x has a terminating decimal if $q = 2^n \times 5^m$
- x has a non-terminating recurring decimal if q has prime factors other than 2 and 5

Classifying 35/50:

35/50 = 7/10 (simplified, HCF = 5)

$10 = 2^1 \times 5^1$ (only 2 and 5 factors)

Terminating decimal

7/10 = 0.7

Classifying 17/6:

6 = 2 × 3 (contains factor 3)

Non-terminating recurring

17/6 = 2.8333... = 2.83

Question 24: Given that HCF(306, 657) = 9, find LCM(306, 657). Also verify using prime factorisation.

Answer:

Using formula:

LCM = (306 × 657) / HCF

LCM = (306 × 657) / 9

LCM = 201042 / 9

LCM = 22338

Verification using prime factorisation:

306 = 2 × 3^2 × 17

657 = 3^2 × 73

HCF = 3^2 = 9

LCM = 2 × 3^2 × 17 × 73

= 2 × 9 × 17 × 73

= 22338

Answer: LCM = 22338

Question 25: Prove that 3 + 2√5 is irrational.

Answer:

Proof by contradiction:

Assume 3 + 2√5 is rational.

Then 3 + 2√5 = p/q where p and q are integers, q ≠ 0, HCF(p,q) = 1.

$2\sqrt{5} = p/q - 3$

$2\sqrt{5} = (p - 3q)/q$

$\sqrt{5} = (p - 3q)/2q$

Since p, q, 3 are integers, $(p - 3q)/2q$ is a rational number.

Therefore our assumption was wrong, and $3 + 2\sqrt{5}$ is irrational.

Formula Table

Concept	**Formula**
Euclid's Division Lemma	$a = bq + r, 0 \le r < b$
HCF × LCM	= Product of two numbers
Terminating decimal condition	$q = 2^n \times 5^m$
Rationalisation	Multiply by conjugate
Sum of irrational	May be rational or irrational

Practice Questions on Real Numbers

1. Identify whether the following numbers are rational or irrational:

- (7)
- (58)
- (π)
- (0.333…)

2. Find the HCF of 36 and 48 using prime factorization.

3. Find the LCM of 20 and 30.

4. Express the decimal number 0.125 as a fraction in simplest form.

5. Find the square root of 196.

196

6. Check whether the number (81) is a real number.

7. Using Euclid's Division Lemma, divide 29 by 5 and find:

- Quotient
- Remainder

$a=bq+r, 0 \leq r<b$

8. Write any five examples of integers and real numbers.

9. Simplify the following:

49+64

10. Determine whether the following statement is true or false:

"Every whole number is a real number."