3. Factors and Multiples

Key Concepts

1. Factors
2. Divisibility rules
3. Types of numbers
4. Chain-splitting method
5. Common factors
6. Highest common factor (HCF)
7. Multiples
8. Common multiples
9. Least common multiple (LCM)

Why should I read this chapter?

Finding factors and multiples of a number is an essential part of Mathematics. They are useful for organising any number of objects into an equal number of groups with an equal number of objects in each group.

Recap

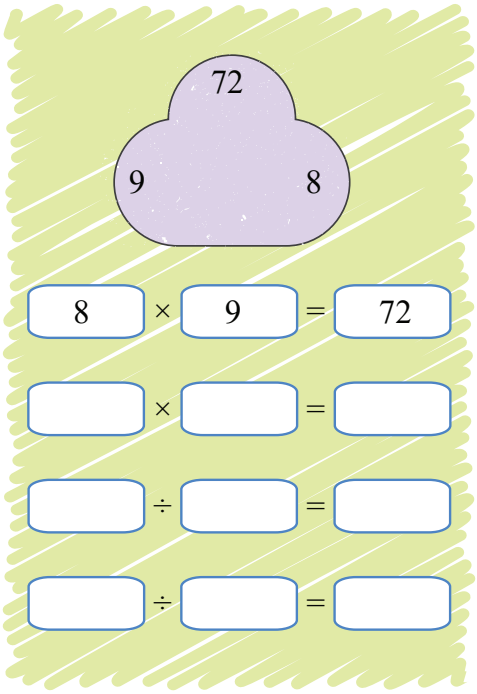
Mohan is an assistant postman at the city post office. He wants to number the red envelopes with the numbers that are divisible by 7.
Help him complete the task by choosing the correct numbers from the number box given below. Write the rest of the numbers on any envelope you like.

56, 46, 98, 63, 80, 114, 133, 85 and 160

Prep-up

1. Use the numbers given below to write multiplication and division sentences. One has been done for you.

a.



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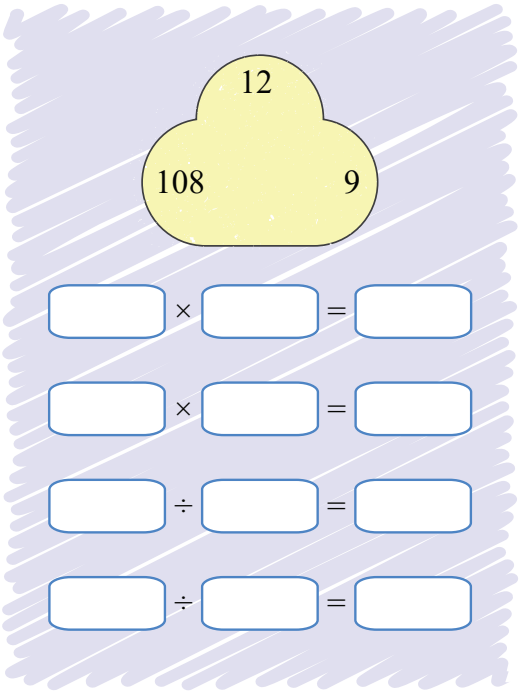
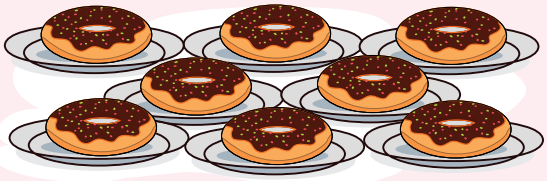


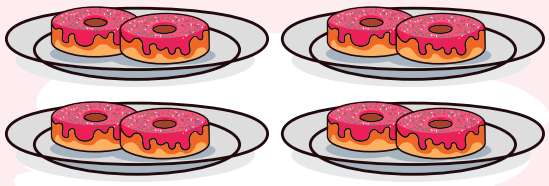
- When two numbers are multiplied to get the product, we can say that each of the numbers can **divide** the product.
- When one number is divided by another number such that the remainder obtained is 0, we can say that the bigger number (dividend) is **divisible by** the smaller number (divisor).

Factors

Rohan, along with seven of his friends, went to buy some doughnuts. They had a few doughnuts and also packed some for their parents. Let us see what they bought.



All of them wanted to have chocolate doughnuts. The baker served 1 chocolate doughnut on a plate to each.
Therefore, the total number of chocolate doughnuts = $1 \times 8 = 8$.

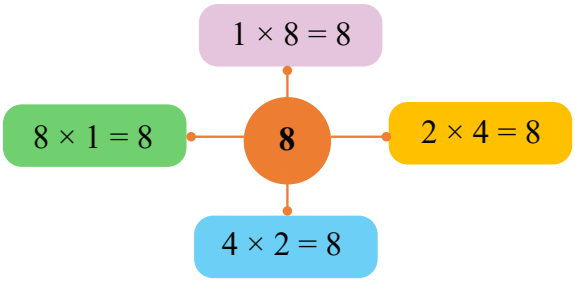


Four of them wanted to have 2 strawberry doughnuts each. The baker served 2 strawberry doughnuts on a plate to each.
Therefore, the total number of strawberry doughnuts = ☐ × ☐ = ☐.

Now, Rohan and Ravi asked the baker to pack 4 orange doughnuts for their parents. The baker packed 4 orange doughnuts in each of the 2 boxes.
Therefore, the total number of orange doughnuts = ☐ × ☐ = ☐.

Priya asked the baker to pack 8 mango doughnuts for her parents. The baker packed the doughnuts in 1 box and gave it to her.
Therefore, the total number of mango doughnuts = ☐ × ☐ = ☐.

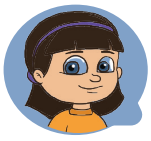
Observe that the total number of each type of doughnut that Rohan and his friends ordered was 8. However, each time, the doughnuts were arranged in different ways.



When we multiply two numbers to get a product, the numbers are known as the **factors** of the product.

Here, 1, 2, 4 and 8 are the **factors** of 8.

A **factor pair** of a number is a pair of factors, which when multiplied gives the number. Here, a factor pair of 8 is 1 and 8. Similarly, another factor pair of 8 is 2 and 4.



Every number has 1 as its factor. The greatest factor of a number is the number itself.

Factors of a number will always be less than or equal to the number.

Example 1: Write the factors of 21.

Solution:

$1 \times 21 = 21$

$3 \times 7 = 21$

$7 \times 3 = 21$

$21 \times 1 = 21$

Factors of 21: 1, 3, 7 and 21

Example 2: Write the factors of 18.

Solution:

$1 \times \square = 18$

$\square \times 9 = 18$

$3 \times \square = 18$

$6 \times \square = 18$

$\square \times 2 = 18$

$18 \times \square = 18$

Factors of 18: $\square$

- We can say that the multiplicand and the multiplier are two factors of their product.
- If dividing a number by another number leaves 0 as the remainder, then the divisor and the quotient are two factors of the dividend.

Progress Meter 1

1. Write all the factors of the given numbers.

(a.) 49 (b.) 26 (c.) 19 (d.) 28 (e.) 35

(f.) 8 (g.) 23 (h.) 38 (i.) 27 (j.) 15

2. Match the following numbers with their factors.

a. 26	○	○	i. 1, 3 and 9
b. 25	○	○	ii. 1, 2, 5 and 10
c. 9	○	○	iii. 1, 2, 13 and 26
d. 4	○	○	iv. 1, 5 and 25
e. 10	○	○	v. 1, 2 and 4

3. Fill in the boxes with the factors of the given numbers.

(a.) 44

$1 \times \square = 44$

$\square \times 22 = 44$

$4 \times \square = 44$

(b.) 32

$\square \times 32 = 32$

$2 \times \square = 32$

$4 \times \square = 32$

(c.) 18

$\square \times 18 = 18$

$2 \times \square = 18$

$\square \times 6 = 18$

(d.) 40

$1 \times \square = 40$

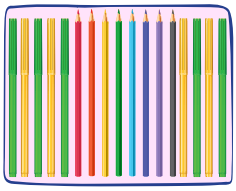
$2 \times \square = 40$

$\square \times 8 = 40$

4. Rohan and Aditya were asked to write all the factors of 84. Rohan wrote 12 factors, whereas Aditya wrote 11 factors. Who wrote the correct number of factors of 84?

Divisibility rules

Rohan bought 72 pens and 85 pencils to distribute among his friends on his birthday. He wants to give each of his friends one pen and one pencil. So, he needs to pack one pen and one pencil in each box.



Will Rohan be able to pack all the pens? ____

How many pencils will remain unpacked? ____

If dividing a number by another number gives 0 as the remainder, then the number (dividend) is said to be **divisible** by the other number (divisor).

Rules of divisibility

2	A number is divisible by 2, if its last digit is 0, 2, 4, 6 or 8.	**Example:** 30, 92, 974, 356 and so on
3	A number is divisible by 3, if the sum of its digits is divisible by 3.	**Example:** 723 = 7 + 2 + 3 = 12 (divisible by 3) 511 = 5 + 1 + 1 = 7 (not divisible by 3)
5	A number is divisible by 5, if the digit in its ones place is 0 or 5.	**Example:** 50, 225, 975 and so on
10	A number is divisible by 10, if the digit in its ones place is 0.	**Example:** 110, 320, 600, 780 and so on

Example 3: Rohan wants to solve the riddle given below. He read the riddle very carefully and chose 540 as the correct answer from the number box. Solve the riddle, and then check whether Rohan was correct.

- I am divisible by 2.
- I am divisible by 3.
- If you divide me by 4, you will get 0 as the remainder.
- I am not divisible by 5.

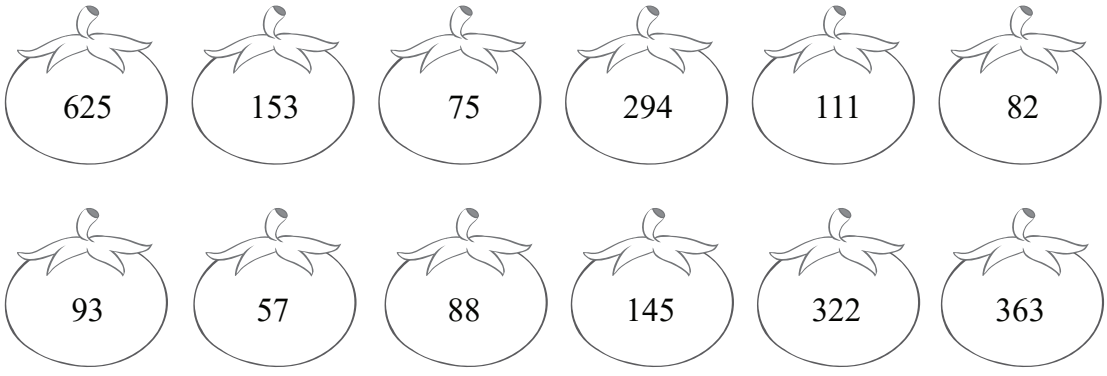
Who am I? _____

324	183	657	540
245	476	751	751

Progress Meter 2

1. Colour the tomatoes with the numbers divisible by 2, 3 and 5 using the given colour code.

divisible by 2	divisible by 3	divisible by 5
red	orange	green



2. Check whether the first number is divisible by the second number using the divisibility rules.

(a.) 172 and 2 (b.) 712 and 5 (c.) 804 and 3 (d.) 374 and 3

(e.) 270 and 10 (f.) 485 and 10 (g.) 285 and 5 (h.) 970 and 2

Types of numbers

64	81	45	89	
37	84	35	21	65
50	22	97	53	25
28	67	74	86	90
46	80	32	73	14
93	15	41	31	38
77	98	56	60	44
59	30	17	45	69
33	54	20	16	

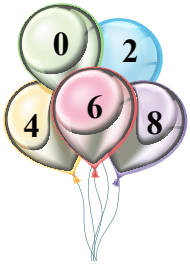
The monkey followed the numbers in green to reach the bananas. Observe the numbers on his path.

Did you notice any difference between the numbers in his path and the rest of the numbers?

The monkey followed _____ (odd/even) numbers to reach the bananas.

Even numbers

The numbers that have 0, 2, 4, 6 or 8 in the ones place are called **even numbers**. An even number is divisible by 2.
Example: 30, 52, 74, 106, 328, 790, 642 and so on

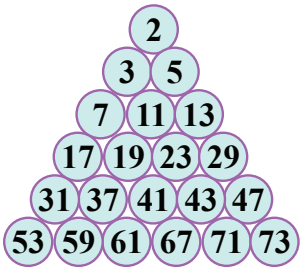


Odd numbers

The numbers that have 1, 3, 5, 7 or 9 in the ones place are called **odd numbers**. An odd number leaves 1 as a remainder when divided by 2.
Example: 41, 63, 95, 227, 309, 677, 231 and so on

Prime numbers

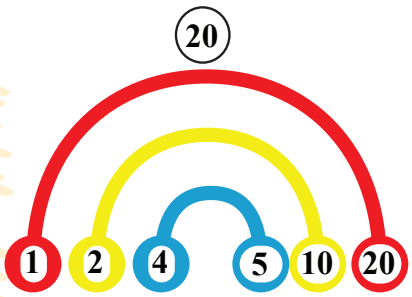
The numbers that have only two factors, 1 and the number itself, are called **prime numbers**.
Example: 2, 3, 5, 7, 11, 13, 17, 19 and so on



Composite numbers

The numbers that have at least one more factor other than 1 and the number itself are called **composite numbers**. Thus, composite numbers have more than two factors.

Example: 20, 88, 91, 27, 66, 12, 34, 51 and so on



- The number 1 is neither a prime number nor a composite number.
- The smallest and the only even prime number is 2. All the other prime numbers are odd.
- The smallest odd prime number is 3.
- The smallest composite number is 4.

Progress Meter 3

1. Write E for even and O for odd numbers.

371	465	165	786	273	599	290	259	258	317
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

242	150	481	876	538	386	675	460	113	550
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

2. Circle all the prime numbers.

12	6	2	14	36	50
33	13	47	24	10	11
34	7	43	19	37	5
45	31	28	23	40	17

3. Circle all the composite numbers.

22	29	46	10	8	53
18	28	44	15	1	16
49	37	33	12	31	9
3	35	20	39	2	21

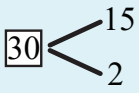
4. Colour the stars with prime numbers yellow and the ones with composite numbers orange.

Chain-splitting method

Example 4: Express 30 as the product of its prime factors.

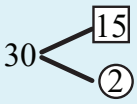
Step 1

Find two factors of the given number.



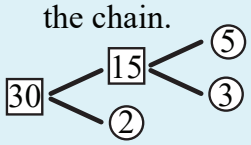
Step 2

Identify the prime factor in the first split in the chain. Circle the prime factor.



Step 3

For the composite number, repeat steps 1 and 2 until you get the prime factors in both the splits of the chain.

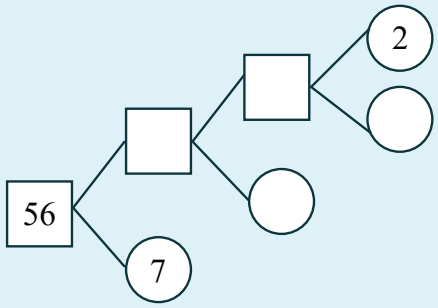


Step 4

Write the given number as the product of all the prime factors obtained.

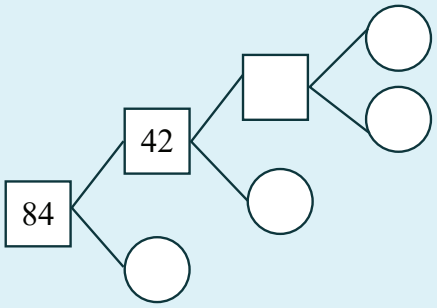
$30 = 2 \times 3 \times 5$

Example 5: Express 56 as the product of its prime factors.



56 = ☐ × ☐ × ☐ × ☐

Example 6: Express 84 as the product of its prime factors.



84 = ☐ × ☐ × ☐ × ☐

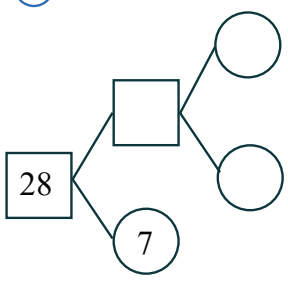
Progress Meter 4

1. Express the following numbers as the product of their prime factors using the chain-splitting method.

(a.) 22	(b.) 24	(c.) 45	(d.) 50	(e.) 25	(f.) 12
(g.) 15	(h.) 21	(i.) 26	(j.) 36	(k.) 78	(l.) 88

2. Write the missing factors.

a.



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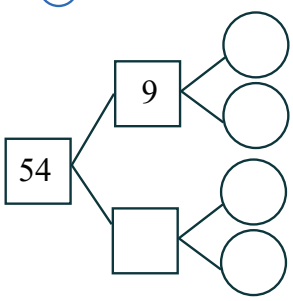
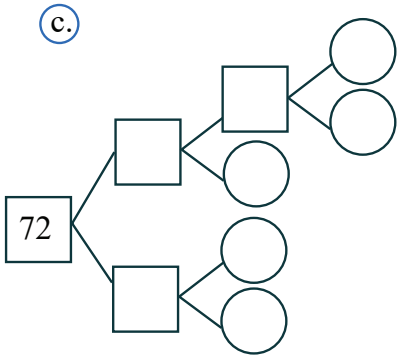


c.



Common factors

Rohan has to identify the factors of 72 and 88 from the given numbers. He has to put a tick(✓) if the given number is a factor and cross(✗) if it is not. Help Rohan complete the table.

Numbers	Factors of 72	Factors of 88
2		
4		
6		
8		
11		

Can you write the numbers which are factors of both 72 and 88?

The factors that are common to two or more numbers are called **common factors** of those numbers.

Example 7: Find the common factors of 72 and 88.

Solution: factors of 72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36 and 72

factors of 88: 1, 2, 4, 8, 11, 22, 44 and 88

common factors: 1, 2, 4 and 8

Example 8: Find the common factors of 75 and 81.

Solution: factors of 75: 1, ☐, ☐, 15, ☐ and 75

factors of 81: 1, ☐, ☐, 27, and 81

common factors: ☐

Progress Meter 5

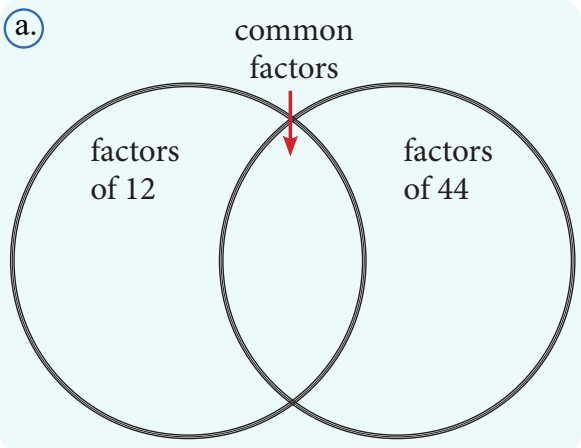
1 . Find the common factors of the following pairs of numbers.

(a.) 16 and 24 (b.) 10 and 22 (c.) 30 and 45 (d.) 7 and 29

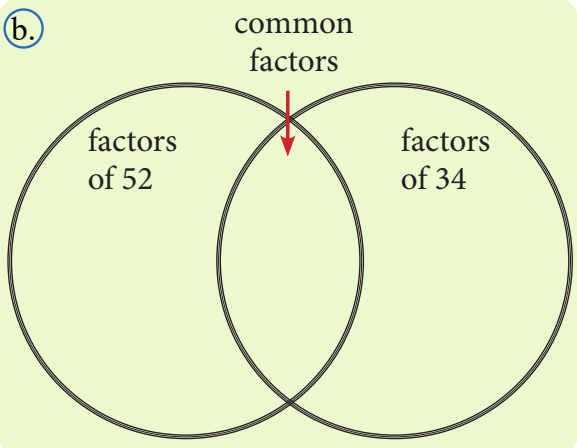
(e.) 45 and 20 (f.) 26 and 9 (g.) 5 and 30 (h.) 28 and 42

2. Write the factors of the given numbers in the circles and the common factors in the middle.

a.



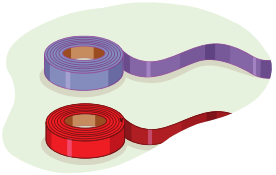
b.



3. Aditya was asked to find the common factors of 32 and 45. He said that the given numbers do not have any factors in common. Is he correct? Justify your answer.

Highest common factor (HCF)

Rohan and Sonia have two rolls of ribbons, which are 42 cm and 63 cm in length, respectively. They have to cut the ribbons in pieces of equal length. What is the greatest length of ribbon they can cut?



To find the greatest equal length into which each of them can cut the rolls of ribbon, we need to find the greatest number that divides both the given numbers (42 and 63).
First, let us list the common factors of 42 and 63.
factors of 42: 1, 2, 3, 6, 7, 14, 21 and 42
factors of 63: 1, 3, 7, 9, 21 and 63
common factors: ____
Which is the greatest among the common factors? ____
Therefore, the greatest number that can divide the lengths of the ribbons Rohan and Sonia have is ____.
The greatest common factor is called the **highest common factor** or **HCF** of the given numbers.

The **highest common factor**, or **HCF**, of two numbers is the greatest number that divides both the numbers.

Example 9: Find the HCF of 30 and 45.
Solution:
factors of 30: 1, 2, ____, 5, 6, ____, 15 and 30
factors of 45: 1, 3, ____, ____, ____ and 45
common factors: 1, 3, ____ and ____
HCF of 30 and 45: ____

HCF is also called the **greatest common divisor**.

Progress Meter 6

1. Find the HCF of the following numbers.

(a.) 10 and 15 (b.) 12 and 20 (c.) 20 and 36 (d.) 14 and 21

(e.) 8 and 12 (f.) 25 and 16 (g.) 15 and 20 (h.) 16 and 18

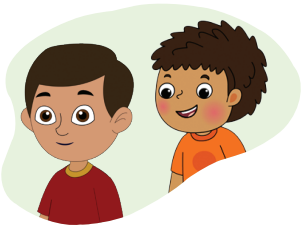
2. Aditya has 84 white papers and 56 coloured papers. He has to arrange the papers in bundles. Each bundle should have an equal and the maximum number of papers. Aditya made bundles of 14. Is it correct? If not, give the correct arrangement of papers.

Multiples

Rohan and Ravi need to complete their holiday homework.

If Rohan solves 5 questions every day, how many questions will he be able to solve in 3 days? ____

If Ravi solves 7 questions every day, how many questions will he be able to solve in 5 days? ____



The numbers 15 and 35 are the **multiples** of 5.

The product obtained on multiplying two numbers is called a **multiple** of both the numbers.

When we multiply 5 with 3, we get 15. Therefore, 15 is a multiple of both 3 and 5.
Similarly, when we multiply 5 with 7, we get 35. Therefore, 35 is a multiple of both 5 and 7.

Every number is a multiple of itself and 1.

The smallest multiple of any number is the number itself.

Example 10: Write the first five multiples of 9.

Solution:

$9 \times 1 = 9$

$9 \times 2 = 18$

$9 \times 3 = 27$

$9 \times 4 = 36$

$9 \times 5 = 45$

Therefore, the first 5 multiples of 9 are 9, 18, 27, 36 and 45.

Example 11: Write the first five multiples of 6.

Solution:

$6 \times 1 = \square$

$6 \times \square = 12$

$6 \times 3 = \square$

$\square \times 4 = 24$

$\square \times \square = 30$

Therefore, the first 5 multiples of 6 are $\square$.

Multiples of a given number are divisible by the number.

Example 12: Check whether 108 is a multiple of 6.

Solution:

To check whether 108 is a multiple of 6, we need to check if 108 is divisible by 6.

$$\begin{array}{r} 18 \\ 6\overline{)108} \\ -\ 6\downarrow \\ \hline 48 \\ -\ 48 \\ \hline 00 \end{array}$$

quotient = 18

remainder = 0

Therefore, 108 is a multiple of 6.

Example 13: Check whether 56 is a multiple of 7.

Solution:

To check whether 56 is a multiple of 7, we need to check if 56 comes in the table of 7.

$56 = 7 \times 8$

Therefore, 56 is a multiple of 7.

A given number can have many multiples.

Progress Meter 7

1. Write the first seven multiples of the following numbers.

(a.) 2	(b.) 7	(c.) 8	(d.) 9
(e.) 11	(f.) 15	(g.) 20	(h.) 17
(i.) 22	(j.) 28	(k.) 30	(l.) 41

2. Do as directed.

(a.) Circle the multiples of 13.

26	92
45	
39	27

(b.) Circle the multiples of 15.

45	150
35	
200	140

(c.) Circle the multiples of 50.

370	90
400	
140	100

(d.) Circle the multiples of 25.

75	95
320	
45	225

3. Check whether the following statements are true or false.

(a.) The number 58 is a multiple of 14.

(b.) The eighth multiple of 12 is 96.

(c.) The number 126 is a multiple of 18.

(d.) The sixth multiple of 16 is 108.

4. Sonia wrote the first six multiples of 12 as given below. Is it correct? If not, then find the error.

multiples of 12: 24, 36, 48, 60, 72 and 84

Common multiples

Write the numbers in the table of 2. ____

Write the numbers in the table of 3. ____

Which numbers are common in the tables of 2 and 3? ____

It means that ____, ____ and ____ are the **common multiples** of 2 and 3.

The multiples common to two or more numbers are called the common multiples of those numbers.

Example 14: Find the common multiples of 4 and 8.

Solution:

multiples of 4: 4, **8**, 12, **16**, 20, **24**, ...

multiples of 8: **8**, **16**, **24**, 40, 48, ...

common multiples: 8, 16, 24, ...

Example 15: Find the common multiples of 6 and 9.

Solution:

multiples of 6: 6, 12, ☐, ☐, ☐, 36, ☐, 48, ☐, ...

multiples of 9: 9, ☐, ☐, ☐, 45, 54, ☐, ☐, ...

common multiples: ☐

Note that these are not the only common multiples of 4 and 8. They have more common multiples.

Progress Meter 8

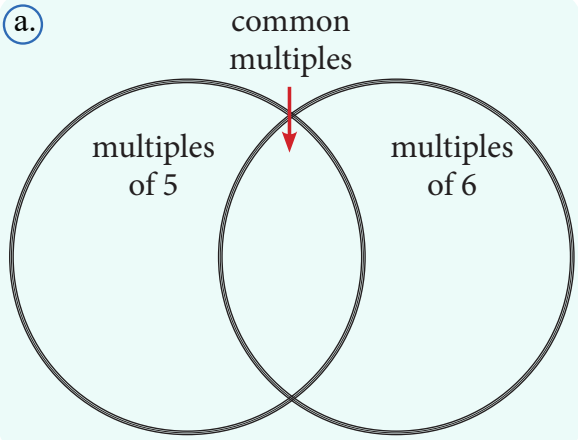
1. Find the first three common multiples of the following pairs of numbers.

(a.) 2 and 6 (b.) 5 and 7 (c.) 2 and 8 (d.) 4 and 12

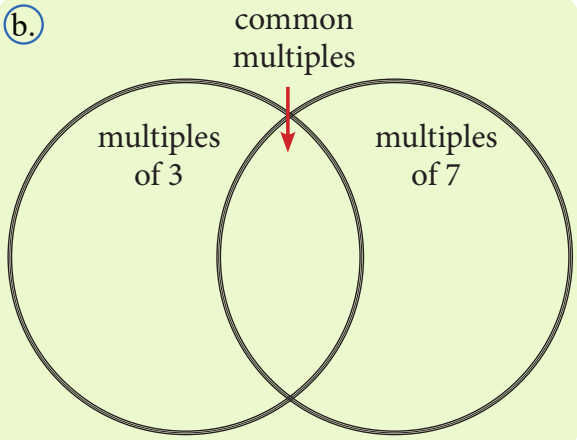
(e.) 3 and 9 (f.) 6 and 14 (g.) 10 and 20 (h.) 13 and 15

2. Write the first ten multiples of the given numbers in the circles and common multiples in the middle.

(a.)



(b.)



Least common multiple (LCM)

Rohan and Aditya attend painting classes. Rohan attends the class every 2 days, while Aditya attends the class every 3 days. Find the day on which both of them will attend the class.

I will attend the class on February 2, 4, 6, 8 and so on.

I will attend the class on February 3, 6, 9, 12 and so on.

Rohan's days of class are in multiples of 2, while Aditya's days of class are in multiples of 3.

multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, ...

multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, ...

common multiples: ____

We see that ____ is the smallest common multiple of 2 and 3.

Rohan and Aditya will have their first class together on February ____.

The smallest among the common multiples of any two numbers is called the **least common multiple (LCM).**

Example 16: Find the LCM of 4 and 5.

Solution:

multiples of 4: ____, ____, ____, 16, 20, ____, 28, 32, ____, ...

multiples of 5: 5, ____, ____, ____, 25, 30, ____, ____, ...

common multiples: 20, ____, ...

LCM of 4 and 5: ____

Progress Meter 9

1. **Find the LCM of the following numbers.**

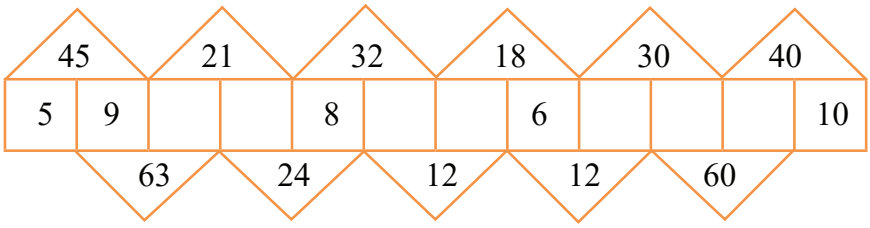
   (a.) 2 and 6 (b.) 9 and 5 (c.) 20 and 36 (d.) 4 and 12

   (e.) 3 and 15 (f.) 4 and 8 (g.) 7 and 9 (h.) 11 and 13

2. **Priya wanted to buy an equal number of patties and buns. If the patties come in packs of 4 and the buns come in packs of 6, then find the least number of packets of each of the patties and buns that Priya should buy to have an equal number of each.**

Mental Maths

1. **The number in each triangle is a common multiple of the numbers in the boxes attached to it. Carefully observe the pattern and fill in the missing numbers.**



2. **The first few multiples of two numbers are written below. Guess the numbers.**

first number		multiples: 18, 24, 30, 36, 42
second number		multiples: 21, 28, 35, 42, 49

3. **Place the correct digits in the boxes to make three different 3-digit numbers that are divisible by 3.**

(a.) ☐ 8 1 (b.) ☐ 8 ☐ (c.) 4 ☐ ☐

Maths Connect

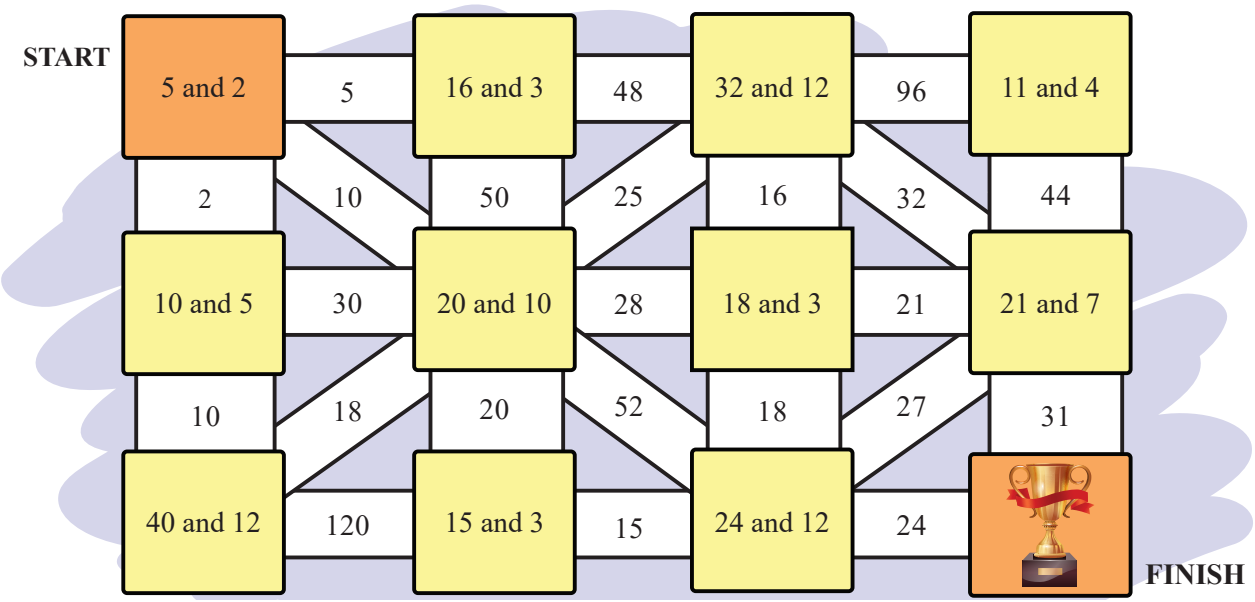
On World Earth Day, three sections, A, B, and C of Grade 4, were given plant saplings. Section A got 4, Section B got 6, and Section C got 8 saplings. The students were expected to plant them in rows such that every row has an equal number of plants. How would they arrange these saplings?

Draw and show the different ways they can plant the saplings. One has been done for you.

Section A	Section B	Section C
		

Fun Time

Find the LCM of the numbers given in the boxes and colour the path showing the correct LCM green to reach the cup.



Exercise

1. Find the factors of the following.

(a.) 22 (b.) 32 (c.) 46 (d.) 75

2. Find the common factors of the following pairs of numbers.

(a.) 38 and 36 (b.) 27 and 30 (c.) 56 and 77 (d.) 48 and 84

3. Find the HCF.

(a.) 48 and 60 (b.) 24 and 36 (c.) 49 and 114 (d.) 86 and 106

4. Find the common multiples.

(a.) 13 and 21 (b.) 14 and 23 (c.) 12 and 25 (d.) 20 and 32

5. Find the LCM.

(a.) 11 and 44 (b.) 15 and 10 (c.) 45 and 85 (d.) 36 and 52

6. Check whether the first number is divisible by the second number.

(a.) 250 and 5 (b.) 495 and 3 (c.) 371 and 2 (d.) 386 and 2

7. Write all the odd numbers between 18 and 38.

8. Write all the prime numbers between 10 and 30.

9. Write all the even numbers between 666 and 677.

10. Write all the prime numbers between 55 and 69.

11. Choose the correct options.

(a.) Which of the following options show all the factors of 8?

i. 2, 4 and 8 ii. 1, 2 and 8 iii. 1, 2, 4 and 8 iv. 8, 16 and 24

(b.) Which of the following is NOT a common factor of 12 and 6?

i. 2 ii. 1 iii. 6 iv. 4

(c.) Write the first three multiples of 7.

i. 7, 14 and 21 | ii. 7, 14 and 23 | iii. 7, 14 and 27 | iv. 1, 7 and 14

(d.) The smallest even prime number is ____.

i. 3 | ii. 4 | iii. 1 | iv. 2

(e.) The smallest composite number is ____.

i. 2 | ii. 3 | iii. 4 | iv. 5

(f.) The number ____ is a factor of every number.

i. 2 | ii. 1 | iii. 0 | iv. 3

(g.) The HCF of 28 and 49 is ____.

i. 8 | ii. 4 | iii. 7 | iv. 14

(h.) Any number, which is divisible by 10 is also divisible by ____.

i. 2, 3 and 5 | ii. 2 and 3 | iii. 2 and 5 | iv. 5 and 6

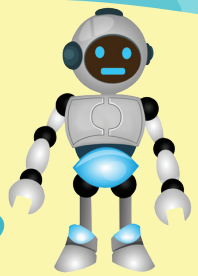
(i.) The LCM of 6 and 5 is ____.

i. 6 | ii. 1 | iii. 5 | iv. 30

(j.) The number 343 is a multiple of ____.

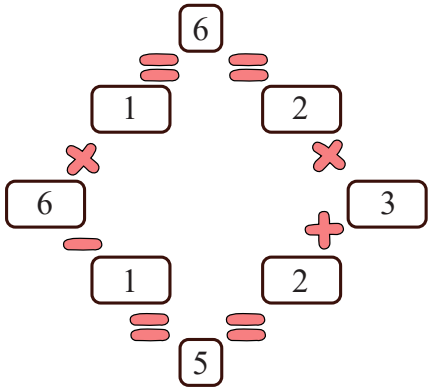
i. 9 | ii. 7 | iii. 5 | iv. 3

Think Class

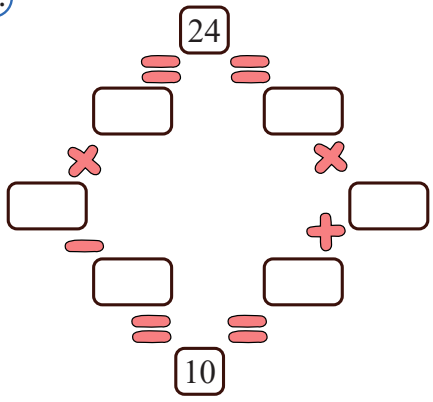


Rohan's robot asked him some questions on factors and multiples. Can you help Rohan solve them?

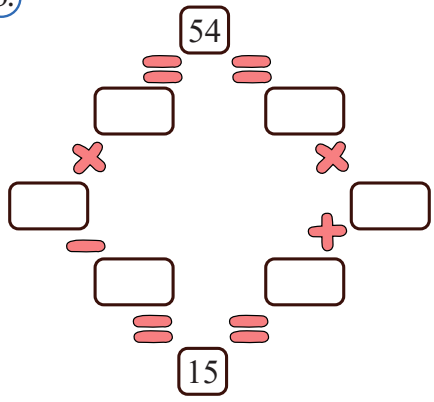
1. Find the factor pairs of the given numbers that make the given sum and difference. One has been done for you.



a.



b.



2. I am a multiple of 70.
I am a number between 200 and 600.
I have odd numbers in the tens place as well as in the hundreds place.
Who am I?

3. Answer the following.

(a.) What is the least number that should be added to 325 to make it divisible by 3?

(b.) What is the least number that should be subtracted from 339 to make the difference divisible by both 3 and 10?