Class 10 Mathematics - Real Numbers

Name: _____

Date: _____

Advanced Worksheet 4

Questions

1. Prove that √7 is irrational.
2. Prove that √11 is irrational.
3. Show that 2 + √3 is irrational.
4. Show that 5√2 is irrational.
5. Show that √2 + √5 is irrational.
6. What method is commonly used to prove the irrationality of numbers such as √2, √3 and √5?

Answer Key

1.

Assume √7 is rational.

Let √7 = a/b where a and b are coprime.

Then $7b^2 = a^2$.

Therefore 7 divides a^2.

Hence 7 divides a.

Let a = 7c.

Substituting gives $b^2 = 7c^2$.

Therefore 7 divides b.

This contradicts the assumption that a and b are coprime.

Hence $\sqrt{7}$ is irrational.

2.

Similar proof using prime number 11.

Hence $\sqrt{11}$ is irrational.

3.

Assume $2 + \sqrt{3}$ is rational.

Then $\sqrt{3}$ = (rational number) − 2.

Thus $\sqrt{3}$ becomes rational, which is a contradiction.

Hence $2 + \sqrt{3}$ is irrational.

4.

Assume $5\sqrt{2}$ is rational.

Then $\sqrt{2}$ = (rational number)/5.

Thus $\sqrt{2}$ becomes rational, contradiction.

Hence $5\sqrt{2}$ is irrational.

5.

Assume $\sqrt{2} + \sqrt{5}$ is rational.

Then $\sqrt{5}$ = (rational number) − $\sqrt{2}$.

Right side becomes irrational-rational contradiction.

Hence $\sqrt{2} + \sqrt{5}$ is irrational.

6.

Proof by contradiction.